



# MULTICRITERION OPTIMIZATION OF THE CUTTING REGIMES IN THE CONDITIONS OF CHANGEABILITY OF THE OPERATING LIMITATIONS

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**Abstract:** The results of the multicriterion optimization of the clean turning regimes of the shaped surfaces - the feed and the cutting speed are presented taking into account the changeability of the operating limitations. Basic criteria of optimization are the maximum productivity, the minimum prime price and multiplicative association of these criteria, which is attitude of the prime price toward the productivity. Methods of the optimization are the linear and geometrical programming. There are set conformities to law of change of the basic variable limitations cutting temperature and the work surface roughness in dependence on position of blade top on the shaped surface. The results of the optimization are set as the analytical dependences of the optimum values of the cutting speed and feed from the parameters of clean turning process of the shaped surfaces taking into account changeability of the operating limitations on the cutting temperature and roughness of the work surface.

**Key words:** turning, shaped surfaces, temperature, roughness, optimization, productivity.

## 1. INTRODUCTION

The task of the increase of the manufacturing efficiency successfully decides based on the optimization of the machining parameters. The methods of the linear and nonlinear optimization of the cutting regimes on the criteria of the maximum productivity and the minimum prime price are well enough developed presently [1, 2]. However with the use of these methods mainly there are decided only the tasks of one criterion optimization of the cutting regimes.

Modern tendencies in optimization are the decision of multicriterion tasks, providing the simultaneous improvement of the machining parameters on the different criteria. One of directions to multicriterion optimization is the use of multiplicative association of the criteria, allowing the different criteria of the optimization to result in the single criterion, which provides the best combination each of them, [3, 4].

The feature of the existent methods of the optimization is constancy both most criteria of the optimization and the operating limitations. However, in a number of cases the parameters of cutting process change appropriately, for example at the machining of the shaped surfaces, in this connection and the

operating limitations become variable. The conformities to law of change of the cutting process parameters at turning of the shaped surfaces: the tool geometrical parameters, the cut section parameters, the chip formation parameters, the cutting forces well enough are studied, [5].

The technique of determining with the use of the linear programming method the optimum on the maximum productivity criterion cutting regimes taking into account the changeability of the cutting processing parameters at turning of the shaped surfaces is presented in [6]. In the work [7] with the use of the geometrical programming method, the optimization of the turning regimes of shaped surfaces on the minimum prime price criterion taking into account the changeability of operating temperature limitations and limitations on durability of cutting plate is carried out.

The purpose of the present work is the perfection of the technique of the multicriterion optimization cutting regimes at the clean turning of the shaped surfaces taking into account the changeability of the operating limitations.

## 2. GENERAL INFORMATION

At the decision of the optimization tasks as the basic optimization criteria variable parts of the productivity  $P$  and prime price  $C$  depending on the cutting regimes are accepted. In the real work as an additional of optimization criterion multiplicative association of these criteria, being a relation of variable parts of prime price and productivity is offered, depending on the cutting regimes  $C_M = P/C$ .

Objective functions, expressing dependence of optimization criteria on the cutting speed  $V$  and the feed  $S$  appear in a kind:

$$\begin{aligned} P &= VS \rightarrow \max \\ C &= V^{-1} S^{-1} + MK_{\ominus}^{-1/mn_t} V^{kv} S^{ks} \\ C_M &= V^{-2} S^{-2} + MK_{\ominus}^{-1/mn_t} V^{kv-1} S^{ks-1} \end{aligned} \quad (1)$$

where  $M = (t_c + A_u/A)(t^{x_v}/C_V K_V)^{1/m}$ ;  $k_v = 1/m - 1$ ;  $k_s = y_v/m - 1$ ;  $A$  - the expenses for 1 minute of the equipment work;  $A_u$  - the expenses for 1 the cutting tool life periods;  $t_c$  - the restoration time of the tool;  $C_V, K_V$  - factors and  $x_v, y_v, m$  - the indexes characterizing degree of influence of depth  $t$ , feed  $S$  and tool life for  $T$  cutting speed  $V$ ;  $K_\Theta = \Theta_o/\Theta$  - coefficient of the necessary decline of the cutting temperature  $\Theta$  at exceeding by it possible level  $\Theta_o$ .

The basic limitations at a finish turning are the limitation on the cutting temperature and the work surface roughness  $R_a$ .

$$\tilde{N}_\Theta V^{n_t} S^{y_t} t^{x_t} \leq \Theta_o; C_R S^{y_r} \leq R_a \quad (2)$$

where  $C_\Theta$  - factor and  $n_t, y_t, x_t$  - the indexes characterizing degree of influence of the speed  $V$ , feed  $S$  and depth  $t$  for cutting temperature;  $C_R$  - factor  $y_r$  - the index characterizing degree of influence feed  $S$  for the work surface roughness;

Graphs of change of relative objective functions to the productivity  $K_P$ , the prime price  $K_C$  and their multiplicative association  $K_M$  depending on cutting speed  $V$  presented on fig. 1.

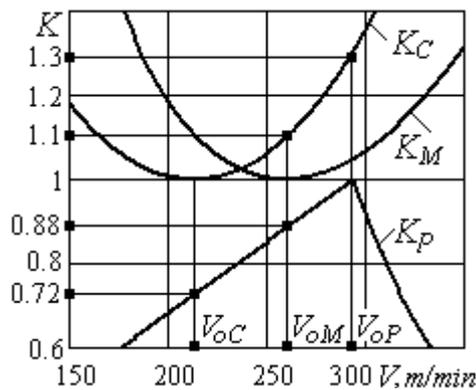


Fig. 1. Graphs of change of relative objective functions the productivity  $K_P$ , the prime price  $K_C$  and their multiplicative association  $K_M$  depending on cutting speed  $V$

Charts are resulted for the followings terms: the finish turning of construction steel by sintered hard alloy cutting tool (work surface roughness  $R_a = 2.5\mu\text{m}$ ).

Charts testify to the presence of the maximum at the function of the productivity  $K_P$  and minimum at the functions of prime price  $K_C$  and their multiplicative association  $K_M$ . Providing a minimum of prime price ( $K_C(V_{oC}) = 1$ ) due to the choice of optimum on prime price of cutting speed  $V_{oC}$ , we get the decline of the productivity in 1,4 time ( $K_P(V_{oC}) = 0.72$ ). Choosing the optimum on the productivity of cutting speed  $V_{oP}$  we provide a maximum of the productivity ( $K_P(V_{oP}) = 1$ ), but get the increase of prime price in 1.3 time ( $K_C(V_{oP}) = 1.3$ ). That providing one extreme level of criteria, we

substantially worsen other. Minimum losses each of objective functions - the productivity ( $K_P(V_{oM}) = 0.88$ ) and prime price ( $K_C(V_{oM}) = 1.1$ ) are arrived at the choice of optimum cutting speed  $V_{oM}$  on multiplicative association of criteria prime price and productivity.

With the use of the linear and geometrical programming method analytical dependences of optimum values of feed and cutting speed from parameters of finish turning are set, [1, 2].

Optimum feed at the finish turning  $S_o$  regardless of the optimization criterion is determined coming from the required work surface roughness  $R_a$ . At presence of temperature limitations cutting speed  $V_{o\Theta}$  is similarly determined regardless of the optimization criterion. The necessity of account of temperature limitations is determined on the basis of border value of coefficient of decline of cutting temperature  $K_{\Theta o} = \Theta_o/\Theta(V_o, S_o)$ .

$$\begin{aligned} S_o &= (R_a/C_R)^{1/y_r} \\ V_{o\Theta} &= (\Theta/C_\Theta t^{x_t} S_o^{y_t})^{1/n_t} \\ K_{\Theta o} &= \Theta_o/V_o^{n_t} S_o^{y_t} t^{x_t} \leq 1 \end{aligned} \quad (3)$$

In default of the temperature, limitations the optimum cutting speeds are determined taking into account the optimization criterion:  $V_{oP}$  - for criterion of the maximum productivity,  $V_{oC}$  - for criterion of the minimum prime price,  $V_{oM}$  - for criterion of the multiplicative association of these criteria.

$$\begin{aligned} V_{oP} &= C_V K_V / T^m t^{x_v} S_o^{y_v} \\ V_{oC} &= (m/(1-m)M)^m S_o^{y_v} \\ V_{oM} &= (2m/(1-2m)M)^m S_o^{y_v} \end{aligned} \quad (4)$$

By the feature of the presented method of the optimization is constancy both most objective functions and the operating limitations. However, in a number of cases the parameters of cutting process change appropriately, for example at the machining of the shaped surfaces (fig. 2), in this connection and the operating limitations become variable



Fig. 2. The detail of metallurgical equipment

At turning of the shaped surfaces, there are variables working edge angle and parameters of cut section along curved surface. Conformities to law of change of these parameters are presented in fig. 3a.

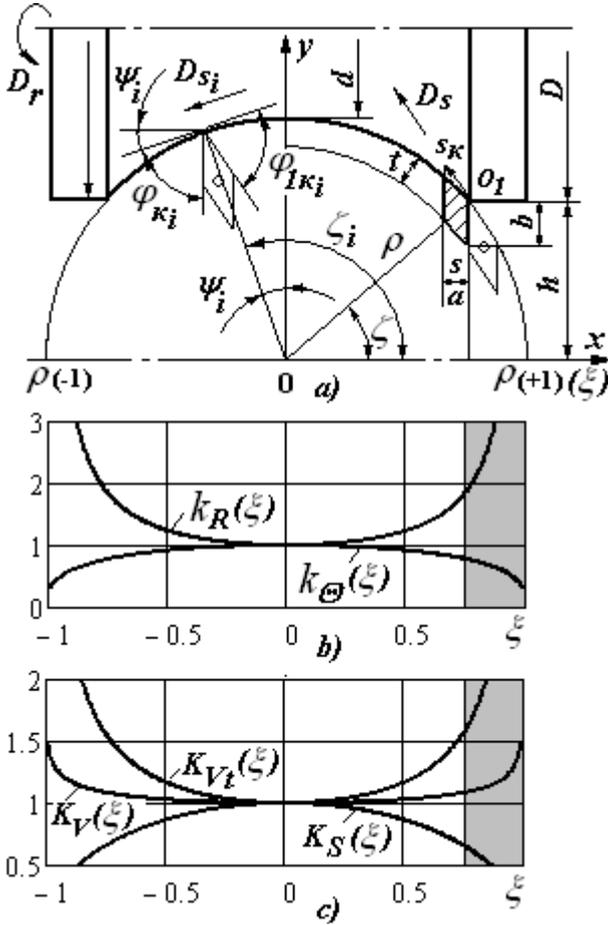


Fig. 3. Charts of change of the dimensionless coefficients  $k_R(\xi)$ ,  $k_\Theta(\xi)$  – b) and  $K_S(\xi)$ ,  $K_V(\xi)$ ,  $K_{Vt}(\xi)$  – c) along a dimensionless co-ordinate  $\xi$  at turning of concave shaped surfaces

Example is resulted for a concave surface (fig. 3a) with the permanent radius of curvature  $\rho$  (generatrix the surfaces of rotation is an arc of circumference, a center of the accepted system of co-ordinates is the center of curvature). Machining is executed by the tool with a rhombic plate for which static working edge angle are  $\varphi_C = 90^\circ$ ;  $\varphi_{1C} = 55^\circ$ . Position of blade top on the indicated curved surface is determined by the instantaneous corner of turn  $\zeta_i = \arccos(\xi_i)$ , where  $\xi_i = x_i/\rho$  is a dimensionless co-ordinate. Working cutting edge angle  $\varphi_K$  and cutting face edge angle  $\varphi_{IK}$  are determined in relation to direction of feed motion  $D_{Si}$  by the  $\psi_i = \arcsin(\xi_i)$ :  $\varphi_{Ki} = \varphi_C - \psi_i$ ;  $\varphi_{IKi} = \varphi_{1C} + \psi_i$ . The feature of machining of concave curved surface is possibility of machining the same cutting tool of surface from the point  $O_1$ , for which  $\varphi_{IK} = 0$ . Maximal possible for machining dimensionless size  $h/\rho = \cos(\varphi_{1C})$ . Shaded area (fig. 3b, c) corresponds to the

area of impossible machining this tool of concave surface.

Geometrical parameters of cut section thickness  $a$ , width  $b$  depend on technological parameters feed  $S$ , cutting depth  $t$  and working edge angle  $\varphi_K$ . At machining of curved surfaces the cutting depth  $t$  remains permanent, in this connection the width of cut  $b$  changes depending only on working cutting edge angle  $\varphi_K$ :  $b(\xi) = t/\sin(\varphi_C - \arcsin\xi)$ .

In the case of machining with a permanent feed on the turn  $S$ , thickness of cut  $a$  remains permanent and a kinematics feed, which coincides with direction of feed motion  $D_S$  is determined  $S_K(\xi) = S/\sin(\arccos(\xi))$ . Because of changeability of the indicated parameters of cutting process at turning of the shaped surfaces basic limitations on the cutting temperature  $\Theta$  and work surface roughness  $R_a$  along curvilinear formative similarly, become variable.

$$R_a(\xi) = C_R k_R(\xi) S^{y_r} \quad (5)$$

$$\Theta(\xi) = C_\Theta k_\Theta(\xi) V^{n_\Theta} S^{y_t} t^{x_t}$$

where  $k_R(\xi)$ ,  $k_\Theta(\xi)$  – dimensionless coefficients characterizing relative change of the work surface roughness  $R_a$  and the cutting temperature  $\Theta$  along a dimensionless co-ordinate  $\xi$  by comparison to the values of these parameters in a point proper to beginning accepted system of co-ordinates  $k_R(\xi) = R_a(\xi)/R_a(0)$ ,  $k_\Theta(\xi) = \Theta(\xi)/\Theta(0)$ .

$$k_R(\xi) = \sin(\arccos(\xi))^{-y_r} \quad (6)$$

$$k_\Theta(\xi) = \sin(\arccos(\xi))^{y_t/n_\Theta}$$

General conformities to law of change of dimensionless coefficients  $k_R(\xi)$ ,  $k_\Theta(\xi)$  along a dimensionless co-ordinate  $\xi$  are presented on fig. 3b.

In connection with changeability of limitations the optimum cutting regimes become also variables along curvilinear formative.

Conformities to law of change of cutting regimes along a dimensionless co-ordinate  $\xi$  are described as follows.

Optimum feed  $S_o$  is determined coming from the required work surface roughness  $R_a$ .

$$S_o(\xi) = K_S(\xi) (R_a/C_R)^{1/y_r} \quad (7)$$

Optimum cutting speed  $V_{ot}$  (taking into account temperature limitations):

$$V_{ot}(\xi) = K_{Vt}(\xi) (\Theta/C_\Theta t^{x_t} S_o^{y_t})^{1/n_t} \quad (8)$$

Optimum cutting speed without the account of temperature limitations:  $V_{oP}$  – for criterion of the maximum productivity,  $V_{oC}$  – for criterion of the minimum prime price,  $V_{oM}$  – for criterion of the multiplicative association of these criteria

$$\begin{aligned} V_{oP}(\xi) &= K_V(\xi) C_V K_V / T^m t^{x_v} S_o^{y_v} \\ V_{oC}(\xi) &= K_V(\xi) (m/(1-m)M)^m S_o^{y_v} \\ V_{oM}(\xi) &= K_V(\xi) (2m/(1-2m)M)^m S_o^{y_v} \end{aligned} \quad (9)$$

where  $K_S(\xi)$ ,  $K_V(\xi)$ ,  $K_{Vt}(\xi)$  – dimensionless coefficients characterizing relative change of the optimum feeds  $S_o$  cutting speeds  $V_o$  along a dimensionless co-ordinate  $\xi$  by comparison to the values of these parameters in a point proper to beginning accepted system of co-ordinates:  $K_S(\xi) = S_o(\xi)/S_o(0)$ ,  $K_V(\xi) = V_o(\xi)/V_o(0)$ ;  $K_{Vt}(\xi) = V_{ot}(\xi)/V_{ot}(0)$ .

$$\begin{aligned} K_S(\xi) &= \sin(\arccos(\xi))^r \\ K_V(\xi) &= \sin(\arccos(\xi))^{-y_v} \\ K_{Vt}(\xi) &= \sin(\arccos(\xi))^{-y_v/n_t} \end{aligned} \quad (10)$$

The set coefficients allow expecting the optimum cutting regimes in any point shaped surface. The examples of calculations of the optimum cutting regimes are resulted for two variants of the machining of the construction steel C45E and corrosion-proof chromium steel X95Cr18 by sintered hard alloy cutting tool (geometrical parameters: major cutting edge angle  $\varphi=90^\circ$ , rake angle  $\gamma=-7^\circ$ ); cutting depth  $t=1\text{mm}$ ; work surface roughness  $R_a=2.5\mu\text{m}$ . The dependences cutting speed  $V$ , cutting temperature  $\Theta$ , work surface roughness  $R_a$  from parameters of cutting process - tool life  $T$ , feed  $S$  and cutting depth  $t$  for construction steel C45E are known [6]:

$$\begin{aligned} V &= 420/T^{0.2} S^{0.2} t^{0.15} \\ \Theta &= 54V^{0.55} S^{0.4} t^{0.2} \\ R_a &= 17.5S^{1.6} \end{aligned} \quad (11)$$

Conformities to law of the cutting temperatures change along a dimensionless co-ordinate  $\xi$  for the indicated terms of themachining, expected for optimum on different criteria to the values of feeds and cutting speeds, are presented on the fig. 4a. In all turndown of cutting temperature does not exceed a possible level  $\Theta_o = 800^\circ\text{C}$ .

Consequently, the optimum cutting regimes are determined without the account of temperature limitations ( $K_{\Theta} \geq 1$ ): the optimum feed  $S_o = 0.3\text{mm/rev}$ ; the optimum cutting speed for criterion of the maximum productivity  $V_{oP} = 293\text{m/min}$ ; for criterion of the minimum prime price  $V_{oC} = 212\text{m/min}$ ; for criterion

of the multiplicative association of these criteria  $V_{oM} = 258\text{m/min}$ . As a result of comparison of these optimum cutting speeds it is set that optimum at prime price cutting speed in 1.4 time is less speed, optimum on the productivity (coefficient of change  $K_{VC} = 0.72$ ); optimum on multiplicative association of criteria cutting speed in 1.1 time is less optimum on the productivity cutting speed (coefficient of change  $K_{VM} = 0.88$ ). In general case the indicated coefficients can be expected:  $K_{VC} = V_{oC}/V_{oP}$ ;  $K_{VM} = V_{oM}/V_{oP}$ .

$$\begin{aligned} K_{VC} &= [mT/(1-m)(t_c + A_u/A)]^m \\ K_{VM} &= [2mT/(1-2m)(t_c + A_u/A)]^m \end{aligned} \quad (12)$$

Conformities to law of change of the optimum cutting speed along a dimensionless co-ordinate  $\xi$  are presented on the fig. 4b. It is set that in the different points of curvilinear surface the optimum cutting speed can change to 1.5 times. It must be taken into account at the choice of the optimum cutting regimes.

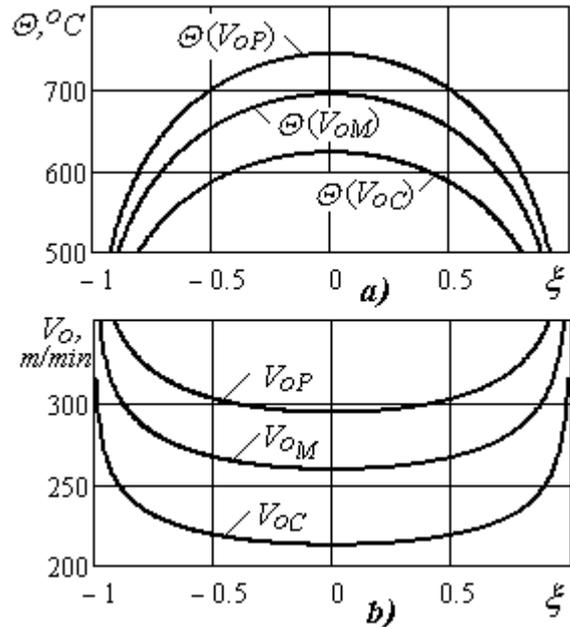


Fig. 4. Charts of change of the cutting temperatures  $\Theta$  change along a dimensionless co-ordinate  $\xi$  – a) and optimum on different criteria cutting speed – b) along a dimensionless co-ordinate  $\xi$  at turning of concave shaped surfaces steel C45E

The second example of the calculation of the optimum cutting regimes is resulted for hard-processing corrosion-proof chromium steel X95Cr18, which is most widespread at making of details of metallurgical equipment. Because of low low heat conductivity of this steel a temperature in the cutting area is considerably higher, than at treatment of construction steels. The dependences cutting speed  $V$ , cutting temperature  $\Theta$ , work surface roughness  $R_a$  from parameters of cutting process - tool life  $T$ , feed  $S$  and cutting depth  $t$

for corrosion-proof chromium steel X95Cr18 are known, [1]:

$$\begin{aligned} V &= 165/T^{0.25} S^{0.15} t^{0.15} \\ \Theta &= 85V^{0.7} S^{0.5} t^{0.3} \\ R_a &= 30S^{1.8} \end{aligned} \quad (13)$$

Conformities to law of the cutting temperatures change along a dimensionless co-ordinate  $\xi$  for the indicated terms of the machining, expected for optimum on different criteria to the values of feeds and cutting speeds, are presented on the fig. 5a. For these regimes cutting temperature exceed a possible level  $\Theta_o = 800^\circ\text{C}$ . Consequently, the optimum cutting regimes are determined with the account of temperature limitations ( $K_{\Theta_o} \leq 1$ ): the optimum feed  $S_o = 0.25\text{mm/rev}$ ; the optimum cutting speed  $V_{oPt} = 55\text{m/min}$ . The account of temperature limitations considerably reduces optimum of cutting speed. The removal of temperature limitations will allow substantially raising the machining productivity. Most effectively it is deciding with use of coated carbide cutting tools and technological cutting fluid, [1].

The optimum cutting speed expected taking into account of temperature limitations considerable below, than the cutting speed expected on different criteria: the optimum cutting speed for criterion of the maximum productivity  $V_{oP} = 103\text{m/min}$ ; for criterion of the minimum prime price  $V_{oC} = 91\text{m/min}$ ; for criterion of the multiplicative association of these criteria  $V_{oM} = 68\text{m/min}$ . The removal of temperature limitations will allow raising the productivity in 1.8 time, to cut prime cost in 1.25 time.

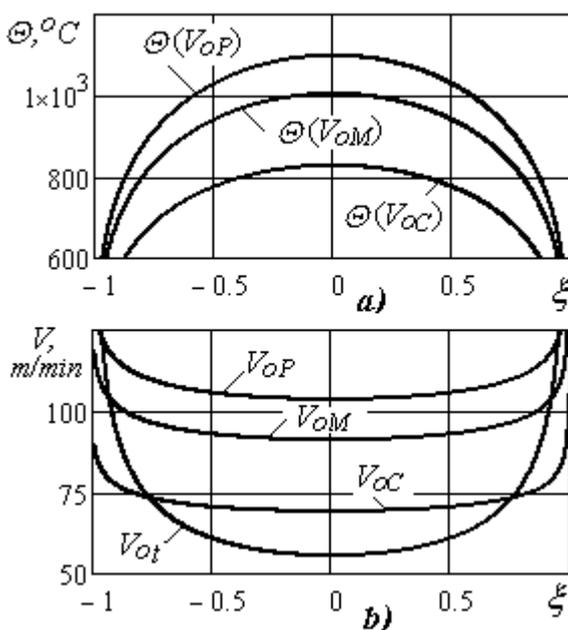


Fig. 5. Charts of change of the cutting temperatures  $\Theta$  change along a dimensionless co-ordinate  $\xi$  – a) and optimum on different criteria cutting speed – b) along a dimensionless co-ordinate  $\xi$  at turning of concave shaped surfaces steel X95Cr18

### 3. CONCLUSIONS

It is improved the technique of multicriterion optimization of the cutting regimes at the finish turning of the shaped surfaces. On the basis of the developed technique the estimation of possibilities of increase of the productivity of hard-processing steels machining to 2 times, declines of prime price in 1.3 time due to the removal of temperature limitations is executed. The results of the optimization are set as the analytical dependences of the optimum values of the cutting speed and feed from the parameters of the finish turning of the shaped surfaces taking into account changeability of the operating limitations on the cutting temperature and roughness of the work surface. It is set that in the different points of curvilinear surface the optimum cutting regimes can change to 1.5 – 2 times and it must be taken into account at the choice of the optimum cutting regimes.

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