SOLVING COUPLED SUPPLY CHAIN PROBLEMS WITH AXIOMATIC DESIGN AND MECHANICAL TOLERANCES DESIGN

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Abstract: This paper explores the likelihood of specifying a supply chain based on an analogy with the tolerancing process in mechanical design and in manufacturing planning. In the case of coupled chains, the specific tolerances for the time span of the different activities depend on the sequence one uses to define the coupled supply chains. In redundant designs, which have coupled chains, the order in which the tolerances are determined is very important. One should begin with the shared links, for which the tolerances should be determined taking into account the chain that requires the tightest tolerances, which is the longest chain for equal delivery tolerances. The Axiomatic Design approach gives a theoretichal confirmation for this method and makes it easy to find out the best sequence to solve the coupled supply chains, especially in the case of multiple chains.

Key words: Supply chain management, tolerancing, Axiomatic Design, redundant designs.

1. INTRODUCTION

There are two key aspects in the tolerancing of mechanical systems: 1) The knowledge about the required funcionality of the system as a whole; 2) the functional analysis of all the components of the complete system.

In systems design, the functionality is usually taken as the nominal response(s) of the system — the output(s). However, the sustained performance depends on the ability of the system to keep the response steady, that is, with variations within a certain range. In tolerance design, the achievement of the functionality of a system is appraised in terms of comparison of the response with the allowed variation, that is, the range in which one can say that the quality of the response is not affected.

In what to the performance of mechanical components corcerns, it is well known that the performance depends on surface quality, thus on the manufacturing tolerances, according to the trend depicted in Fig. 1. It is also known that the relationship between the manufacturing costs and the tolerances can usually be represented by a curve of the same type (Fig.2).

The evolution of both curves can also be explained in terms of human behaviour. In fact, an empirical study (Campos & Nóbrega 2009) shows that the tolerance zone gets closer and higher when the importance of the attributes increases. An approximate mathematical representation of those curves is usually achieved through exponential functions with negative exponents.

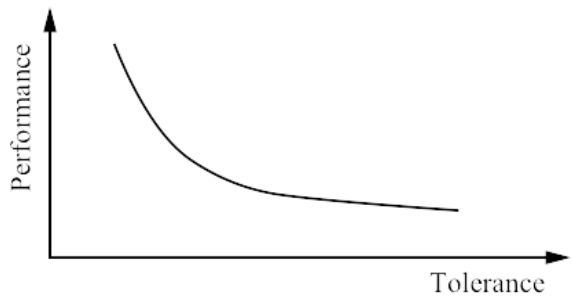

Fig. 1. Qualitative relationship between performance and tolerance

From the combination of both curves, one can conclude: i) a high operational performance implies a high manufacturing cost; ii) systems with loose manufacturing tolerances do not ensure a high operational performance.

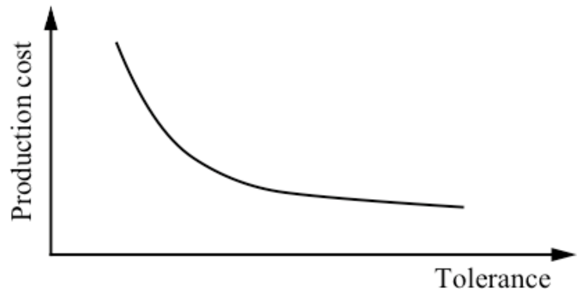

Fig. 2. Qualitative relationship between production costs and tolerance

Beyond this reality, engineers must always keep in mind another truthful statement: for the same manufacturing difficulty (cost), the variation of the operational performance increases with the value of the response. This means that for the same class of tolerance, the higher is the value of the nominal dimension, the higher is the value of the tolerance.

In addition, in mechanical tolerance design one has to determine the minimum chain of dimensions that is required to define the positions of the functional surfaces of the system (the interfaces).

The use of a mechanical tolerancing analogy for the design of supply chains must comply with the above-described concepts, as pointed out by Garg *et al.* (2006) and Choudhary *et al.* (2006). Incidentally, mechanical engineering provides many analogies that can be used in the design of supply chains, namely fluid dynamics (Romano, 2009).

Each actual dimension of a manufactured mechanical system always has a variation comparatively to its nominal value. The maximum allowed variation must be specified, so that the system can reach the prescribed performance. Similarly, the actual activity durations in a supply chain undergo variations relatively to their nominal values. Therefore, the design of supply chains entails the determination of the maximum allowed variation for the elapsed times of all the relevant activities. Failing to do so would make unpredictable both the performance and the operational costs of the supply chain under design.

2. THE DIMENSIONS CHAINS AND THE SUPPLY CHAINS

The first step in mechanical tolerance design is to define the minimal chain of dimensions, all of which are of functional type. In a supply chain, a functional dimension is a basic activity that adds value to the whole operation, as specified. Therefore, the minimal chain of dimensions in a supply chain is the minimum number of activities that one can define and schedule at each decision level.

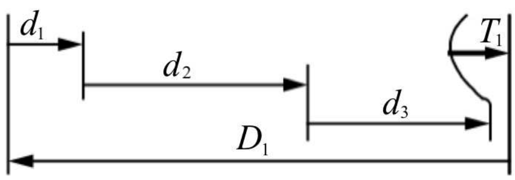

Fig. 3. Simple supply chain

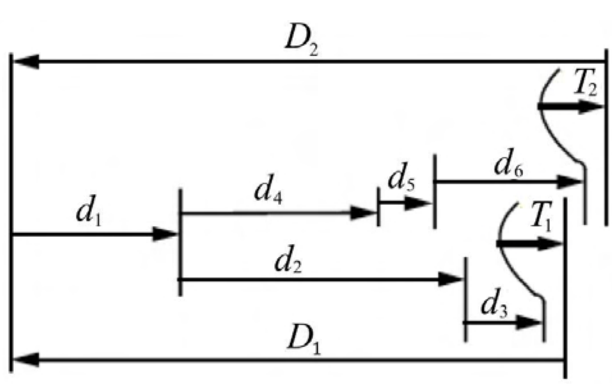

Fig. 4. Coupled supply chains

Like dimension chains, supply chains can be either simple (Fig. 3) or coupled (Fig. 4).

On this issue, Islam (2004) observed, "A body of literature proposing a number of strategies has been published for the solving of these independent functional equations but there has been less interest in the solving of coupled functional equations [a/n: coupled supply chains], which require an extra strategy to determine an optimum solution order". This paper deals with two possible strategies: Design of Mechanical Tolerances and Axiomatic Design.

3. MODELS FOR TOLERANCE ANALYSIS

This paper will consider only the most representative models for the analysis of the tolerances: the worst-case model and the statistical model.

The worst-case model is represented by

$$T = \sum_{i=1}^{n} t_i \qquad (1)$$

where T is the tolerance of the assembly as a whole, t_i is the tolerance of the components, and n is the number of components.

The most usual statistical model (Spotts, 1978) is depicted by

$$T = Z\left[\sum_{i=1}^{n}\left(t_i/z_i\right)^2\right]^{1/2} \qquad (2)$$

where Z is the number of the standard deviations (σ) that is assumed for the dispersion of the delivery time of the supply chain, and z_i is the number of σ_i of the elapsed time dispersion of each activity. Gaussian distributions with $Z = z_i = 6\,\sigma_i$ yield to a simple statistical model.

Both the worst-case and the statistic model can be used in the analysis of supply chains, but the decision between them depends on:

- The number of components of the system (n);
- The quantity of systems to be produced;
- The knowledge on the probability density function of the variations of each dimension and of the whole system.

4. SYNTHESIS METHODS

The tolerance design of mechanical systems basically consists in distributing the assembly tolerance among the system components. Similarly, the design of supply chains would consist in sharing out the delivery tolerance to the distinct activities. This can be achieved through two different approaches: 1) the proportional method; 2) methods of optimization based on cost functions.

According to the proportional method, in supply chains one should start by allocating tolerances to the distinct activities on a basis of the knowledge about nature of the activities or by prerequisite, as it usually happens with outsourced activities. The obtained values are used to check if the prescribed delivery tolerance can be attained. If changing the allocated tolerances is required (except for the prerequisite tolerances), then in the worst-case one should compute a proportionality factor, p_f,

$$p_f = \frac{D_t - \sum_{j=1}^{m} t_j}{\sum_{i=1}^{n-m} t_i} \quad (3)$$

where D_t is the delivery tolerance, t_j are the prerequisite tolerances and t_i are the tolerances to establish.

When the tolerances follow normal distributions with $Z = \zeta_i$, then the following statistic model should apply:

$$p_f = \left(\frac{D_t^{\,2} - \sum_{j=1}^{m} t_j^2}{\sum_{i=1}^{n-m} t_i^2} \right)^{1/2} \quad (4)$$

In both models, fixing the initial tolerance values should be achieved though

$$t = p_f t_i \quad (5)$$

The optimization methods that support tolerance design make use of cost tolerance functions, C_t, which try to describe specific instances of the generic curve depicted in Fig. 2. The most usual functions are:

- Exponential function (Speckhart, 1972)

$$C_t = \sum_{i=1}^{n} \left(A_i + B_i e^{-C_i/t_i} \right) \quad (6)$$

- Reciprocal square function (Spotts, 1973)

$$C_t = \sum_{i=1}^{n} \left(A_i + B_i / t_i^2 \right) \quad (7)$$

- Sutherland function (Sutherland & Roth, 1975)

$$C_t = \sum_{i=1}^{n} \left[A_i + B_i \left(t_i^2 \right)^{-C_i/2} \right] \quad (8)$$

- Michael-Siddal function (Michael & Siddal, 1981)

$$C_t = \sum_{i=1}^{n} \left(A_i + B_i t_i^{-C_i} e^{-D_i t_i} \right) \quad (9)$$

- Reciprocal function (Chase & Greenwood, 1988)

$$C_t = \sum_{i=1}^{n} \left(A_i + B_i / t_i \right) \quad (10)$$

where C_t is the cost tolerance, and the coeficients A_i, B_i, C_i and D_i can be determined from data about each type of activity: A_i depends on fixed costs, while B_i, C_i and D_i depend on variable costs.

5. A CASE STUDY

5.1 The problem

Our case study is depicted by Fig. 4, which describes the processing of two orders for different quantities of the same product that should start at the same time. For a matter of simplicity, the figure only shows the final deadline tolerances, T_i. Finding the time-limit tolerances of each activity is the aim of this section.

Order #1 must arrive to the customer premises within 38 ± 2 days, and will be shipped through outsourced TIR.

Order #2 must debark at an overseas destination port within 40 ± 2 days.

The procurement and provisioning of raw materials will be made at the same time for both orders since they refer to the same product, and will be processed in parallel.

All the activities will be done in such a way that the manufactured products will stay at the fabrication premises as short as possible.

Fig. 4, shows that the orders are coupled trough the common procurement and provisioning activity (d_1).

The other activities of Order #1 are fabrication (d_2) and TIR transportation (d_3). The total time to accomplish Order #1 is T_1.

As for Order #2, after procurement and provisioning (d_1) we have fabrication (d_4), transfer by road to the embarkation port (d_5) and ship delivery to the destination port (d_6). The total time to deliver Order #2 is T_2.

5.2 The mechanical tolerances design method

On the design of mechanical tolerances viewpoint, one can notice that three different kinds of tolerances are considered in our example: prerequisite tolerances, which are enacted by the customers, therefore cannot be changed; empirical tolerances, which are based on empirical experience from previous operations of the same kind; and computed tolerances, which are calculated according to some criteria as a means to allow attaining the enacted time span for the whole operation.

Since the chains for both orders are coupled, the specific toreances for the time span of the different activities depend on the sequence one uses to define the coupled supply chains.

Tables 1 and 2 relates to Case #1, in which we begin by defining the supply chain for Order #1.

Table 1. The supply chain of Order #1 – Case #1

(Units = day)	D_1	d_1	d_2	d_3
Delivery time	38	14	20	4
Prerequisite tolerance	4.00			
Empirical tolerance			1.00	0.60
Computed tolerance, Eq.(1)		2.40		

Table 2. The supply chain of Order #2 – Case #1

(Units = day)	D_2	d_1	d_4	d_5	d_6
Delivery time	40	14	17	1	8
Prerequisite tolerance	4.00	2.40			1.00
Empirical tolerance			0.85	0.20	
2.40+0.85+0.20+1.00 = 4.45 > D_2, Eq. (1)					
p_f =0.571, Eq. (3)					
Allocated tolerance		2.40	0.49	0.11	1.00

In this case, the delivery time for Order #1 (the shorter chain) is easy to attain, but the empirical tolerances that belong to Order #2 must be tightened (because $p_f < 1$), so that the delivery time of Order #2 can be attained.

Tables 3 and 4 depict Case #2, and show that the it is better to begin by defining the longer supply chain (Order #2), because in this circumstance we have $p_f > 1$.

Table 3. The supply chain of Order #2 – Case #2

(Units = day)	D_2	d_1	d_4	d_5	d_6
Delivery time	40	14	17	1	8
Prerequisite tolerance	4.00				1.00
Empirical tolerance			0.85	0.20	
Computed tol., Eq.(1)		1.95			

Table 4. The supply chain of Order #1 – Case #2

(Units = day)	D_1	d_1	d_2	d_3
Delivery time	38	14	20	4
Prerequisite tolerance	4.00	1.95		
Empirical tolerance			1.00	0.60
1.95+1.00+0.60 = 3.55 < D_1, Eq. (1)				
p_f =1.281 , Eq. (3)				
Allocated tolerance		1.95	1.28	0.77

5.3 The Axiomatic Design method

Axiomatic Design (AD is a theory introduced in the early 1970s that aims at supporting decision-making in engineering design (Suh 1990).

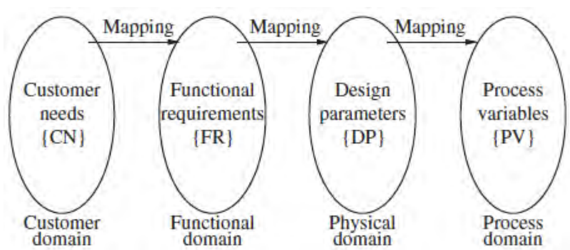

Fig. 5. The Design Domains (adapted from Suh, 1990)

According to AD, any design object, being it a product, aprocess or any other technical system, can be described by a vector in each one of four design domains, as shown in Fig. 5.

The design process usually starts in the customer domain, where the customer needs (CNs) are perceived. Mapping to the functional domain allows finding the functional requirements (FRs). Next, another mapping translates the FRs into design parameters (DPs), *i.e.*, the set of properties that physically describe the design object. At last, mapping from the physical to the process domain leads to the process variables (PVs), *i.e.*, the outline of how to put together the physical components of the design object.

Any single mapping between a pair of contiguous design domains (see Fig. 6) can be represented by the design equation

$$\{Y\} = [A]\{X\}; \quad A_{ij} = \frac{\partial Y_i}{\partial X_j}; \quad i = 1, ..., m; \quad j = 1, ..., n \quad (11)$$

where vector $\{Y\}$ is the set of m requirements that should be accomplished, $\{X\}$ is the vector representing the set of n parameters that is expected to fulfil the requirements, and $[A]$ is the so-called design matrix.

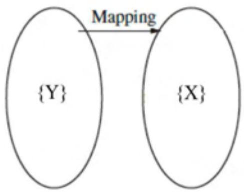

Fig. 6. A typical mapping between contiguous design domains

Eq. (11) is not unique, which means that different $\{X\}$ vectors would represent different design solutions that are characterized by distinct design matrices. The pattern of those matrices would make the difference between "good" and "poor" design, taking into account the Independence Axiom, which states that in good design, the selected parameters $\{X\}$ should be chosen so that the requirements $\{Y\}$ are fulfilled in an independent manner. Thus, the ideal design solution should have the same number of requirements and parameters ($m = n$) and the design matrix should be diagonal, case of which the design solution is called "uncoupled". A triangular design matrix is also acceptable and corresponds to a "decoupled" design. The name "decoupled" is given because the design parameters can be adjusted in an independent manner if the right sequence is followed. Any other pattern for a square design matrix corresponds to a "coupled" design, which should be recognized as poor and as such should be avoided (Suh, 1990).

For any design solution where $m > n$, AD's Theorem 1 states that either the design is coupled or some of its FRs can never be fulfilled (Suh, 1990). In the case of $m < n$, AD's Theorem 3 states that the design is either redundant or coupled (Suh, 1990), and Coelho *et al.* (2011) showed that a design with $m < n$ is redundant if its design matrix is trapezoidal.

Since Eq. (11) is not unique, it is most likely to find distinct solutions that are acceptable for the same set of FRs, case of which the Information Axiom provides a quantitative means for evaluating the relative merit of such design solutions. This axiom states that in a set of design solutions that satisfy the same FRs and conform to the Independence Axiom, the best is the one with the minimum information content. The information content, I, for the simple case of a design with only one FR and one DP is defined as being the inverse of the probability of achieving the design goal with a random (but possible) value for the DP:

$$I = \log_2 \frac{(\textit{area of the system range})}{(\textit{area of the common range})}, \quad (12)$$

where the area of the system range is computed from the FR's probability density function, and the area of the common range is the fraction of the above-mentioned area that is inside of the design range limits (see Fig. 7).

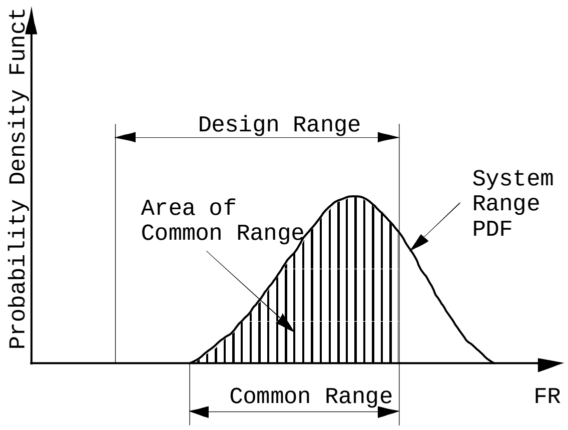

Fig. 7. The distinct ranges of a design

For an uncoupled design with n FRs, the total information content, I_t, can be computed through

$$I_t = \sum_{i=1}^{n} -\log_2 p_i = \sum_{i=1}^{n} I_i, \quad (13)$$

where p_i is the probability of FR_i being satisfied by DP_i.

In an AD point of view, the supply chain of Fig. 4 can be described by

$$\begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{Bmatrix} \quad (14)$$

where $\{D_i\}$, $i = 1, \ldots, n$, denotes the "vector of the design requirements", and $\{d_j\}$, $j = 1, \ldots, m$, is the "vector of the design parameters".

In terms of tolerances, Eq. (14) can be rewritten as

$$\begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{Bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \end{Bmatrix}, \quad (15)$$

that is, the prerequisite tolerances T_1 and T_2 for the final deadlines are used to establish the tolerances δ_j for the time extent of the diffent activities, t_j.

Eq. (15) also shows that

$$\begin{cases} T_1 = t_1 + (t_2 + t_3) \\ T_2 = t_1 + (t_4 + t_5 + t_6) \end{cases} \quad (16)$$

which means that the following simplification is valid

$$\begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} d_1 \\ (d_2 + d_3) \\ (d_4 + d_5 + d_6) \end{Bmatrix} \quad (17)$$

As a rule, summing parameters like we have done in Eq. (17) is not allowed. However, supply chains is an exception because the time extent of both the activities and of their tolerances have the same substantive meanings and are expressed in the same physical units (time).

Because Eq. (17) has $m > n$ and its design matrix is trapezoidal, one concludes that Fig. 4 represents a redundant design (Gonçalves-Coelho *et al.* 2011). Even better, one can see that de design is uncoupled because the 2x2 block at the far right of the design matrix is diagonal. This means that the only coupling between Order #1 and Order #2 is the task of duration d_1, and that we could start determining the tolerances of any one of the chains. Yet, we should choose the path of minimum information content, as per AD's Information Axiom.

The information content of each tolerance is given by

$$I_j = \log_2 \left(\frac{d_j}{t_j} \right), \quad (18)$$

which shows that the larger is the time extent of an activity, the larger shoud be its tolerance, as a means to ensure a low information content. On the other hand, Eq. (16) shows that the larger is the supply chain, the smaller should be the sum of its tolerances, t_j. But Eq. (16) also shows that if we begin with Order #1, then we will achive a larger value for t_1, which implies smaller values for t_4, t_5 and t_6, thus making larger the information content of Order #2. Therefore, in this case one should begin with Order #2, as it was done in Tables 3 and 4.

6. DISCUSSION AND CONCLUSIONS

Wu & ElMaraghy (1988) have performed a series of Monte Carlo computer simulations with different methods of synthesis and analysis for mechanical tolerancing, with different tolerance distributions and different number of components in the dimensions chain. Some relevant conclusions that could be obtained from those simulations are the following:

-The reliability of the methods other than the worst-case is low if the number of the chain components is small and their tolerance values are very different.

-The results obtained through statistic models are highly dependent on the probability density functions that are considered for the tolerances of the components' dimensions.

In what to the synthesis methods concern, some of them just relate to the achievement of suitable allocation of tolerances, while others also take into account cost optimization. However, the benefit that results from cost optimization may lead to flawed technical solutions if there is not sufficient knowledge about the processes that are involved in the activities.

Therefore, the crucial point of any supply chain is the definition of its configuration.

According to what we have seen, coupled chains should be avoided and simple chains should be as short as possible, in order to allow tolerances as loose as possible for each one of their links.

In the case of coupled chains, the order in which the tolerances are determined is very important. One should begin with the shared links, for which the tolerances should be determined taking into account the chain that requires the tightest tolerances, which is the longest chain for equal delivery tolerances.

By analogy with mechanical design, several approaches could be adopted in order to attain better performance from supply chains:

-Focusing in the structure of the supply chain: the supply chain should be as short as possible.

-The supply chain should be designed in a concurrent engineering environment.

-An analogy with "design for X" should be adopted, which means selecting the operations with the maximum probability of success.

-The knowledge about the involved processes should be improved in order to allow estimating costs, elapsed times and probability density functions.

In the lack of sufficient knowledge, the worst-case method should be adopted. As a rule, this is the case of novel supply chains.

The Axiomatic Design approach gives a theoretichal confirmation for all the aforesaid conclusions and makes it easy to find out the best sequence to solve the coupled supply chains, especially in the case of multiple chains.

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