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END MILL TOOL'S PROFILING – GRAPHICAL SOLUTION IN CATIA, USING THE GENERATING TRAJECTORIES METHOD

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Abstract: The helical cylindrical surface, with constant pitch, generation (screws with multiple starts; helical flutes of the cutting tools or some worms from gears) may be made using end mill tools. These tools' types may be profiled based on the theory of reciprocally enwrapping surfaces. A method based on a complementary theorem from the domain of surface enveloping - the method of generating relative trajectories - is proposed in this paper. The method is developed as application in CATIA design environment. Application examples of the graphical algorithm are presented and the results are compared with those obtained by analytical solutions in order to validate the proposed method quality.

Key words: surface enveloping, end mill, graphical profiling method.

1. INTRODUCTION

The profiling of tools with primary peripheral revolution surfaces edges, for the generation of constant pitch helical surfaces (helical gears, helical pumps screws, helical compressor rotors of driven screws with multiple starts) is made using the fundamental principles of surface enwrapping - the Olivier theorem or the Gohman theorem [1]. Moreover, other dedicated theorems, such as the Nikolaev theorem [2], or complementary theorems: the "substitution circles family" or "minimal distance" [3] may be used for this specific situation of helical surface generation using tools with revolution surfaces edges. At the same time, the development of graphical design environment such as AutoCAD, CATIA and others have allowed to approach this issue of determining the shape of reciprocally enwrapping revolution surfaces using graphical methods [4, 5].

The issue of helical surfaces generation using tools with revolution surfaces edges - side mills; end mills or ring tools - is a current problem, as shown by recent research and publications [6-12].

This paper proposes a solution for the profiling of the end mill, a tool edged by a primary peripheral revolution surface, based on the knowledge of trajectories described in the relative motion between the surface to be generated and the peripheral surface of the future tool, both in analytical and graphical expressions - the method of relative generation trajectories.

Regarding the specific issues of generation with an end mill tool, a tool edged by a primary peripheral revolution surface, the problem may be simplified by analysing the contact between the two conjugated surfaces - the helical cylindrical surface with constant pitch and the revolution surface, as an in plane problem, in plane sections orthogonal to the axis of the revolution surface, see Figure 1.

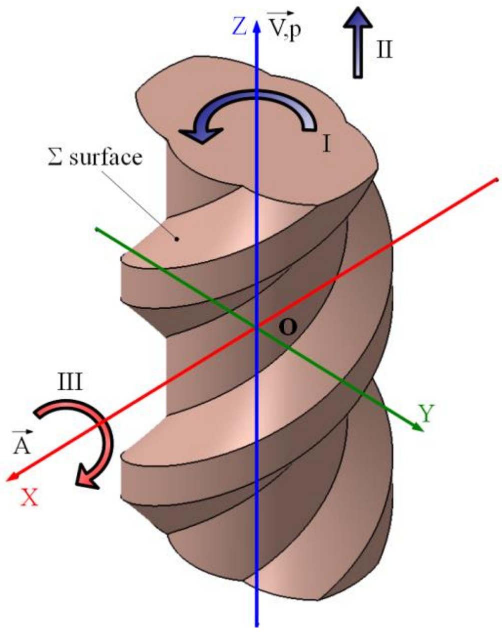

Fig. 1. The process kinematics

The generation process kinematics assumes to know the movements:

I is the rotation of the helical surface around its own axis, $\vec{V}$ axis;

II - the translations of the helical surface along its own axis, linked with the rotation motion *I* so as to generate a helicoid with a *p* parameter, identical with the helicoid to which the $\sum$ helical surface belongs;

III - the rotation of end mill tool around its own axis, the $\vec{A}$ axis.

In the *I* and *II* set of motions, the $\sum$ helical surface, as a cylindrical helical surface with constant pitch, with $\vec{V}$ axis and *p* helical parameter, is self-generated. The characteristic curve of the $\sum$ surface, in the *I*, *II* and *III* set of motions does not depend on the motion component during which it is self-generated [2].

Thus, the characteristic curve of the $\sum$ surface will be determined only by its rotation motion around the axis of the future end mill tool, the $\vec{A}$ axis. In the transverse *H* plane, see Figure 2, the parallel circle of the revolution surface result as enwrapping of the trajectories family, the family of $C_{\Sigma H}$ curves, in the rotation movement around the $\vec{A}$ axis.

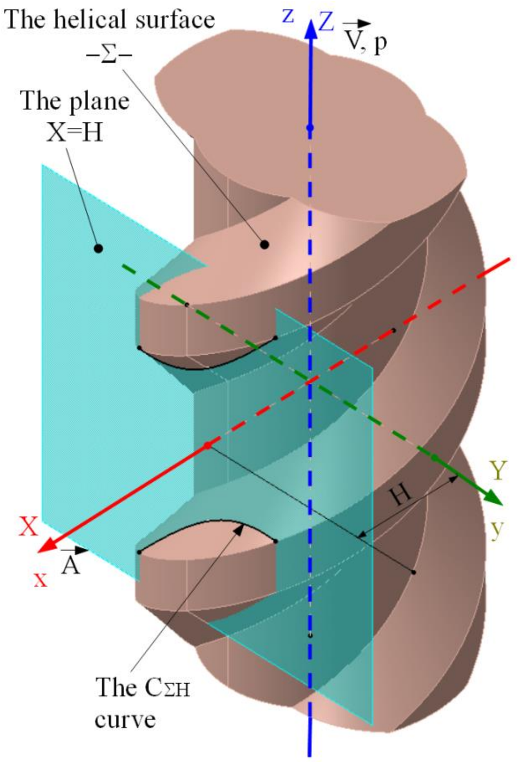

Fig. 2. Helical surface; the plane *X=H*;
the reference systems

2. RELATIVE GENERATION TRAJECTORIES FAMILY METHOD

The reference systems joined with the helical surface that generates the model of the $\sum$ helical surface and the in-plane orthogonal to the axis of the future revolution surface - the primary peripheral surface of the end mill tool are presented in Figure 2.

The reference systems are defined:

xyz is the fixed reference system, with the *x*-axis is overlapped with the $\vec{A}$ axis of the future end mill tool. The $\vec{A}$ axis intersects the *z*-axis, the axis of the $\sum$ helical surface.

XYZ - is the mobile reference system joined with the helical surface, the $\vec{V}$ overlapped with the *Z*-axis in the initial position, overlapped with the *xyz* reference system. If it is accepted that the helical surface admits a generatrix defined as:

$$G\left| \begin{aligned} &X = X(u); \\ &Y = Y(u); \\ &Z = Z(u), \end{aligned} \right. \tag{1}$$

with the *u* variable parameter, in its helical motion around the $\vec{V}$ axis, with the *p* helical parameter,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X(u) \\ Y(u) \\ Z(u) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ p\cdot\varphi \end{pmatrix} \tag{2}$$

worm with right-hand flighting, we obtain the parametrical form of the helical surface equations, in the *XYZ* reference system:

$$\Sigma\left| \begin{aligned} &X = X(u)\cos\varphi - Y(u)\sin\varphi; \\ &Y = Y(u)\sin\varphi + X(u)\cos\varphi; \\ &Z = Z(u) + p\cdot\varphi, \end{aligned} \right. \tag{3}$$

with the φ angular parameter, for the rotation around the *Z*-axis. The orthogonal plane to the $\vec{A}$ axis, of the future end mill tool, Figure 2, is defined:

$$X = H \text{ (H is arbitrary variable)}, \tag{4}$$

or, from (3):

$$X(u)\cos\varphi - Y(u)\sin\varphi = H. \tag{5}$$

The condition (5) is, in principle, equivalent with the form:

$$\varphi = \varphi(u, H). \tag{6}$$

The set of equations (3) and (6) defines, on the helical surface, an in-plane curve $C_{\Sigma H}$. In principle, the $C_{\Sigma H}$ curve is expressed by equations in the form:

$$C_{\Sigma H}\begin{vmatrix} X=H; \\ Y=Y(u); \\ Z=Z(u), \end{vmatrix} \quad (7)$$

for a variable value of the *H* parameter.

The $C_{\Sigma H}$ curves, from various planes *X=H* (*H* variable), in the rotation movement around the $\vec{A}$ axis, will describe a family of generation trajectories, with the enwrapping as the primary peripheral surface of the end mill tool. The relative generating trajectories method is based on a specific definition and, at the same time, on a specific theorem, as follows:

Definition: The relative generating trajectories are defined as the trajectories generated in the rotation motion around the axis of the revolution surface conjugated with the points belonging to the curves on the helical surface, from the in plane sections, orthogonal to the axis of the conjugated revolution surface.

Theorem: The revolution surface conjugated with a cylindrical helical surface with constant pitch is the geometric locus of the enwrapping of generation trajectories, in the rotation motion around the axis of the revolution surface.

2.1 The enwrapping condition

The determination of the enwrapping condition specific to the proposed generation problem assumes to find the normal at the $C_{\Sigma H}$ curves, with its parameters determined by:

$$\vec{N}_{C_{\Sigma H}}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & \dot{Y}_u & \dot{Z}_u \\ 1 & 0 & 0 \end{vmatrix} \quad (8)$$

or, in other form,

$$\vec{N}_{C_{\Sigma H}}=\dot{Z}_u\cdot\vec{j}-\dot{Y}_u\cdot\vec{k} \quad (9)$$

The form (9) represents a vector with the direction of the normal line versor at the curve $C_{\Sigma H}$, in the *XYZ* system. The supporting line of the normal line at the $C_{\Sigma H}$ curve has the equations:

$$N_{C_{\Sigma H}}\begin{vmatrix} X=H; \\ Y=Y(u)+\lambda\cdot\dot{Z}_u; \\ Z=Z(u)-\lambda\cdot\dot{Y}_u, \end{vmatrix} \quad (10)$$

with the λ variable scalar parameter and *H* discrete arbitrary variable. Now, the $N_{C_{\Sigma H}}$ family of straight lines can be determined, in their rotation motion around the $\vec{A}$ axis (the axis of end mill tool):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos v & -\sin v \\ 0 & \sin v & \cos v \end{pmatrix}\cdot\begin{pmatrix} H \\ Y(u)+\lambda\cdot\dot{Z}_u \\ Z(u)-\lambda\cdot\dot{Y}_u \end{pmatrix} \quad (11)$$

or, after developments:

$$\begin{cases} x=H; \\ y=\left[Y(u)+\lambda\cdot\dot{Z}_u\right]\cdot\cos v-\left[Z(u)-\lambda\cdot\dot{Y}_u\right]\cdot\sin v; \\ z=\left[Y(u)+\lambda\cdot\dot{Z}_u\right]\cdot\sin v+\left[Z(u)-\lambda\cdot\dot{Y}_u\right]\cdot\cos v. \end{cases} \quad (12)$$

with the *v* angular parameter of rotation around the *x*-axis. The equations (12) represent the family of normal lines at $C_{\Sigma H}$, generated in the rotation motion around the $\vec{A}$ axis. From the condition that the normal lines at the $C_{\Sigma H}$ curves to be identically with the normal lines at the future peripheral revolution surface of the end mill tool, the condition is imposed that the family of normal lines (12) intersects the $\vec{A}$ axis, the axis of the revolution surfaces, namely the axis of end mill tool.

The $\vec{A}$ axis equation can be described, in the xyz reference system, in form:

$$x=H;\ y=0;\ z=0, \quad (13)$$

H - scalar variable measured along the *x*-axis. Thus, from (12) and (13), the following conditions result:

$$\begin{aligned} &\left[Y(u)+\lambda\cdot\dot{Z}_u\right]\cdot\cos v-\left[Z(u)-\lambda\cdot\dot{Y}_u\right]\cdot\sin v=0; \\ &\left[Y(u)+\lambda\cdot\dot{Z}_u\right]\cdot\sin v+\left[Z(u)-\lambda\cdot\dot{Y}_u\right]\cdot\cos v=0. \end{aligned} \quad (14)$$

out of which, by eliminating the λ parameter, the following specific enwrapping condition results:

$$Y(u)\cdot\dot{Y}_u+Z(u)\cdot\dot{Z}_u=0. \quad (15)$$

In principle, the condition (15) represents an algebraic link between the *u* and *v* parameters, defining a geometric locus on the ∑ surface. The geometric locus signifies the characteristic curve of the ∑ surface in the *I*, *II* and *III* set of movements, see Figure 1. We reiterate the observation that, in the *I-II* composed motion, only the component that refers to the rotation around the axis of the end mill tool has influence on the form of the characteristic curve. We note that from (12), for $\lambda=0$, the C_{Σ} family of curves is determined in its rotation movement around the $\vec{A}$ axis, the axis of the end mill tool, the family of relative generation trajectories:

$$\left(C_{\Sigma}\right)_v \begin{vmatrix} x = H; \\ y = Y(u)\cdot\cos v - Z(u)\cdot\sin v; \\ z = Y(u)\cdot\sin v + Z(u)\cdot\cos v, \end{vmatrix} \qquad (16)$$

which, together with the condition (15), represent the characteristic (contact curve) between the ∑ helical surface to be generated and the revolution surface *S*.

2.2 Axial section of the *S* surface

The enwrapping condition (15) represents a link between the variable parameters *u* and *v*:

$$v = v(u), \qquad (17)$$

thus, in principle, the equations (15) have analytical forms which depend to one single parameter:

$$C_S \begin{vmatrix} x = H; \\ y = Y(u); \\ z = Z(u), \end{vmatrix} \qquad (18)$$

the C_S curve being a curve belonging to the *S* surface, see Figure 3.
The axial section is expressed by:

$$S_A \begin{vmatrix} H = x; \\ R = \sqrt{Y^2(u) + Z^2(u)}. \end{vmatrix} \qquad (19)$$

3. APPLICATIONS

3.1 End mill tool for a square thread

An example for relative generation trajectories method application is proposed in connection with the profiling of end mill tool, which generates a worm with rectilinear flanks, with multiple starts. These types of screws are used as driving elements for slider of the mechanical press. The reference system to which the worm in linked, as well as the axial section of the helical flanks - straight line in planes orthogonal to the axis of the worm to be generated are presented in the Figure 4.

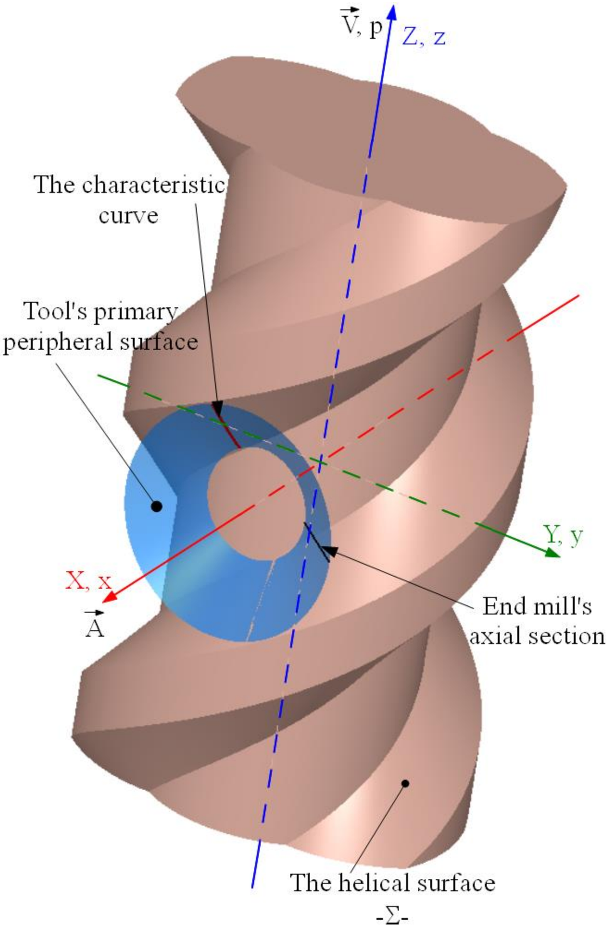

Fig. 3. Characteristic curve and axial section of the end mill tool

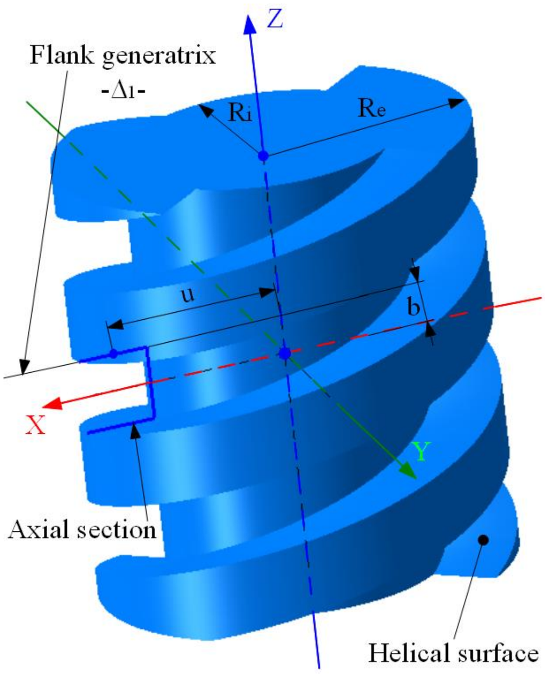

Fig. 4. Helical surface. Thread with *k* starts and the flank generatrix Δ_1

The generatrix of the helical flank is give by:

$$\Delta_1 \begin{vmatrix} X = u; \\ Y = 0; \\ Z = b, \end{vmatrix} \quad (20)$$

with $u_{min} = R_i$; $u_{max} = R_e$ constructive values. The $\sum$ helical surface, a helix with $\vec{V}$ axis and p helical parameter, has the equations:

$$\Sigma \begin{vmatrix} X = u \cdot cos\,\varphi; \\ Y = u \cdot sin\,\varphi; \\ Z = b + p \cdot \varphi. \end{vmatrix} \quad (21)$$

The C_{Σ} curve, is defined in plane:

$$X = H \quad (22)$$

(H variable between limits $H_{min}=R_i$ and $H_{max}=R_e$).
From (21) and (22) results:

$$u = \frac{H}{cos\,\varphi}. \quad (23)$$

Thus, the $C_{\Sigma H}$ curves, see also (7), have form:

$$C_{\Sigma H} \begin{vmatrix} X = H; \\ Y = \dfrac{H \cdot sin\,\varphi}{cos\,\varphi}; \\ Z = b + p \cdot \varphi. \end{vmatrix} \quad (24)$$

The derivatives from (24) are defined in order to calculate the enwrapping condition (15):

$$\begin{aligned} \dot{Y}_{\varphi} &= \frac{H}{cos^2\,\varphi}; \\ \dot{Z}_{\varphi} &= p. \end{aligned} \quad (25)$$

The set of equations of type (16):

$$\begin{aligned} X &= H; \\ Y &= Y(\varphi) \cdot cos\,v - Z(\varphi) \cdot sin\,v; \\ Z &= Y(\varphi) \cdot sin\,v + Z(\varphi) \cdot cos\,v, \end{aligned} \quad (26)$$

with $Y(\varphi)$, $Z(\varphi)$ give by (24) and condition (15), calculated for derivatives (25), with H arbitrary variable, represents the primary peripheral surface of the end mill tool.
A numerical application is presented for profiling the end mill tool designed to generate a worm with characteristics: R_e = 60 mm; R_i = 40 mm; number of starts k = 2; b = 20 mm.

They are proposed graphical solutions developed in CATIA design environment. The $\sum$ surface is modeled and, knowing the axis position of the future end mill, they are defined orthogonal planes on this axis.
The intersection of these planes (INTERSECTION command) with the 3D model of the $\sum$ surface determines the C_{Σ} curve. The generating trajectories family in the $X=H$ plane is obtained revolving the C_{Σ} curve around the tool's axis, see Figure 5.
Next, a circle tangent to the generating family is constructed.
This circle represents a parallel circle of the revolution surface (the primary peripheral surface of the tool), Figure 6.
The intersection of this circle with the C_{Σ} curve, obtained with command INTERSECTION, represents a point onto the characteristic curve.
The characteristic curve is common for helical surface of the blank and, at the same time, for revolution surface of the tool, Figure 7a and b. Revolving (REVOLVE command) the characteristic curve around the X axis is generated the S surface.

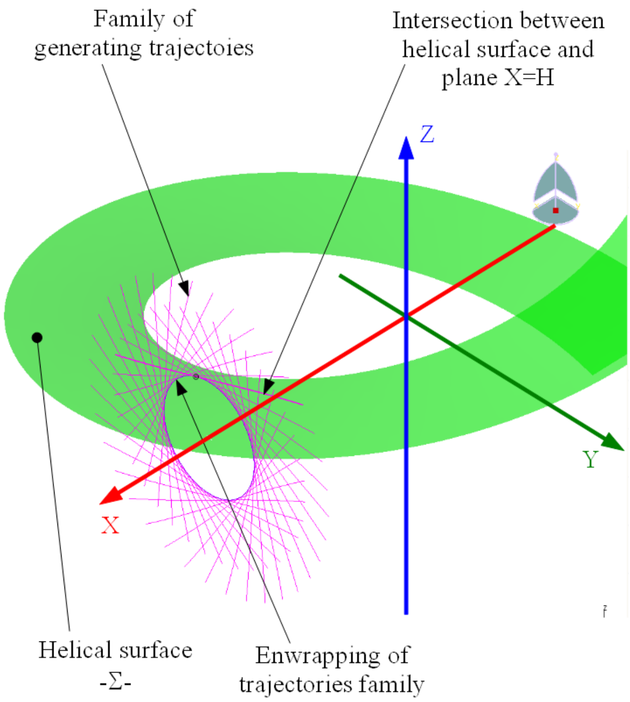

Fig. 5. Intersection between helical surface and planes; family of C_{Σ} curves; enwrapping of family

The section with an axial plane (INTERSECTION command) determines the form of the axial section and it is possible to measure coordinates of points which belong to this section (MEASURE command).
In Table 1 are presented the coordinates of point from the characteristic curve and the error calculated as value of the specific enwrapping condition, see equation (15).

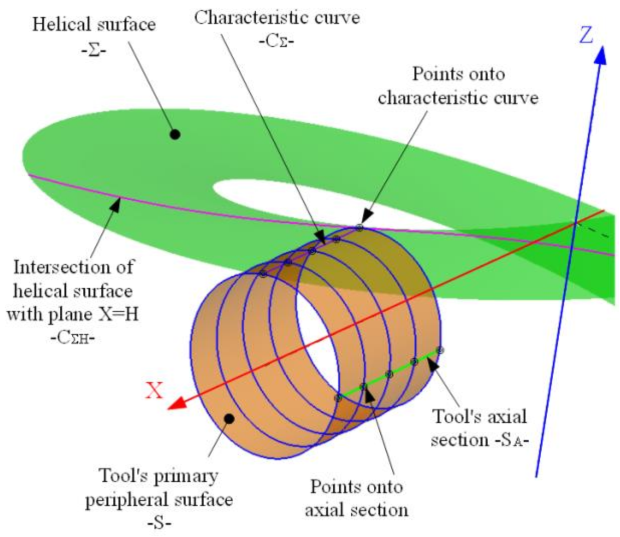

Fig. 6. Tool's primary peripheral surface; characteristic curve; axial section

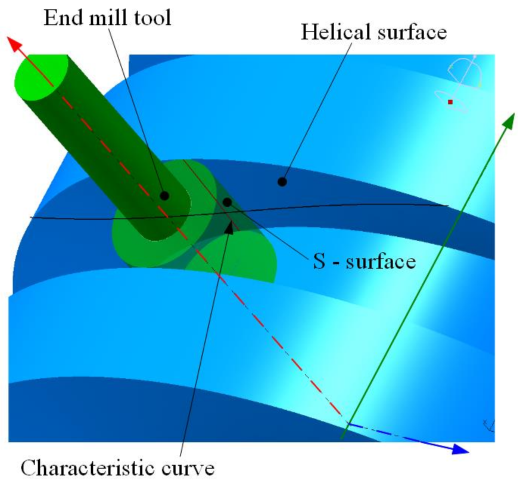

Fig. 7a. Characteristic curve

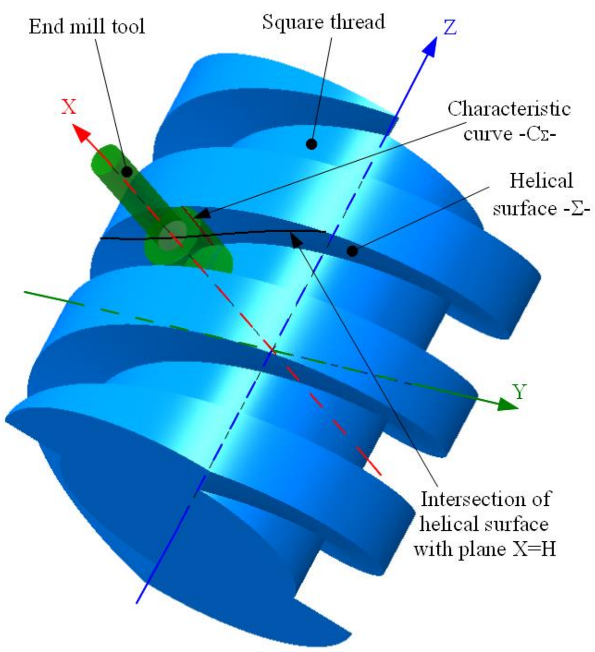

Fig. 7b. Tool's working position

Table 1. Coordinates of points from the characteristic curve

X [mm]	Y [mm]	Z [mm]	Error
60.000	-2.029	9.570	0.000
58.000	-2.029	9.514	0.003
56.000	-2.159	9.509	0.004
54.000	-2.230	9.474	0.005
52.000	-2.306	9.436	0.005
50.000	-2.386	9.393	0.006
48.000	-2.472	9.345	0.007
46.000	-2.564	9.291	0.009
44.000	-2.661	9.231	0.010
42.000	-2.766	9.163	0.012
40.000	-2.877	9.086	0.015

The form of the end mill tool's axial section and coordinates of points belong to this section are presented in Table 2 and Figure 8.

Table 2. Coordinates of points from the axial section

R [mm]	H [mm]	R [mm]	H [mm]
60.000	9.783	50.000	9.691
58.000	9.754	48.000	9.666
56.000	9.751	46.000	9.638
54.000	9.733	44.000	9.607
52.000	9.714	42.000	9.571

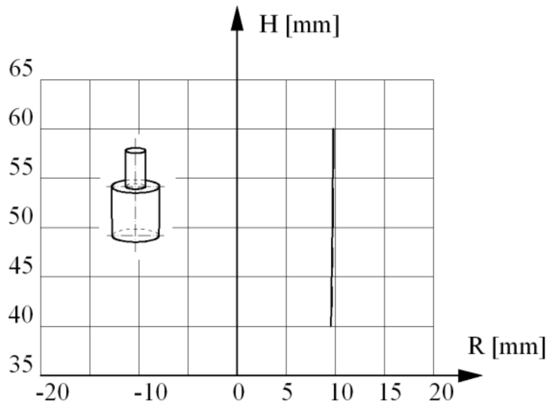

Fig. 8. Axial section of end mill tool

3.2. End mill tool for an involute worm

The involute worm definition is known as ruled surface [1].

The reference systems as generators and the circle with the R_b radius - the cross section of the base cylinder are presented in Figure 9.

The generatrix of the involute worm is defined as the straight-line Δ, tangent to the cylinder with R_b radius and, at the same time, tangent to the helix described by the intersection point between the Δ line and the cylinder with R_b radius:

$$\Delta \left| \begin{array}{l} X_0 = R_b; \\ Y_0 = u \cdot \sin \chi; \\ Z_0 = u \cdot \cos \chi. \end{array} \right. \qquad (27)$$

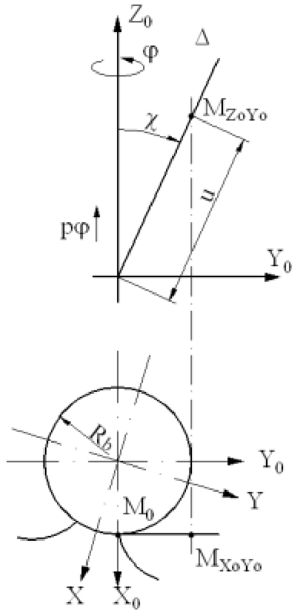

Fig. 9. Reference systems and crossing section of the involute worm

In the helical motion $\vec{V}$ and p (with $\vec{V}$ overlapped to the Z_0 axis) described in form:

$$\begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} R_b \\ u\cdot\sin\chi \\ u\cdot\cos\chi \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ p\cdot\varphi \end{pmatrix}, \quad (28)$$

with φ variable parameter, a ruled surface is generated. The surface equations are:

$$(\Delta)\left| \begin{aligned} &X_0 = R_b\cdot\cos\varphi - u\cdot\sin\chi\cdot\sin\varphi; \\ &Y_0 = R_b\cdot\sin\varphi + u\cdot\sin\chi\cdot\cos\varphi; \\ &Z_0 = u\cdot\cos\chi + p\cdot\varphi, \end{aligned} \right. \quad (29)$$

with notation

$$tg\chi = \frac{R_b}{p}. \quad (30)$$

For a worm with k starts, the equations (29) re reported to a new reference system, XYZ, see Figure 9. The half angle of gap is δ and the X axis is the symmetry axis of the gap between two anti-homologous flanks:

$$\delta = \frac{m\cdot\pi}{4}\cdot\frac{2}{m\cdot k} = \frac{\pi}{2\cdot k}. \quad (31)$$

Thus, by transformation of the coordinates:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos\delta & -\sin\delta & 0 \\ \sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}, \quad (32)$$

the equations (29) are transferred to the reference system which admits X axis as symmetry axis of the gap:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos\delta & -\sin\delta & 0 \\ \sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} R_b\cdot\cos\varphi - u\cdot\sin\chi\cdot\sin\varphi \\ R_b\cdot\sin\varphi + u\cdot\sin\chi\cdot\cos\varphi \\ u\cdot\cos\chi + p\cdot\varphi \end{pmatrix} \quad (33)$$

or

$$\Sigma = (\Delta)_\varphi \left| \begin{aligned} &X = R_b\cdot\cos(\delta+\varphi) - u\cdot\sin\chi\cdot\sin(\delta+\varphi); \\ &Y = R_b\cdot\sin(\delta+\varphi) + u\cdot\sin\chi\cdot\cos(\delta+\varphi); \\ &Z = u\cdot\cos\chi + p\cdot\varphi. \end{aligned} \right. \quad (34)$$

The $C_{\Sigma H}$ curve, see (7) is deduced from condition:

$$H = R_b\cdot\cos(\delta+\varphi) - u\cdot\sin\chi\cdot\sin(\delta+\varphi) \quad (35)$$

or, otherwise,

$$u = \frac{\left[R_b\cdot\cos(\delta+\varphi) - H\right]}{\sin\chi\cdot\sin(\delta+\varphi)}. \quad (36)$$

For determining of enveloping condition, they are calculated:

$$\begin{aligned} &\dot{Y}_\varphi = R_b\cdot\cos(\delta+\varphi) + \dot{u}_\varphi\cdot\sin\chi\cdot\cos(\delta+\varphi) - \\ &-u\cdot\sin\chi\cdot\sin(\delta+\varphi); \\ &\dot{Z}_\varphi = \dot{u}_\varphi\cdot\cos\chi + p; \end{aligned} \quad (37)$$

and

$$\dot{u}_\varphi = \frac{-R_b + H\cdot\cos(\delta+\varphi)}{\sin\chi\cdot\sin^2(\delta+\varphi)}. \quad (38)$$

The set of equations (35) and (34), see also (37) and (38), together with condition (15), determines the characteristic curve onto the $\sum$ surface. The axial section of the S surface is deduced from (34):

$$S_A: \left| \begin{aligned} &X = H;\ (\text{H-variable, } H_{min} = R_b,\ H_{max} = R_e); \\ &R = \sqrt{Y^2(\varphi) + Z^2(\varphi)}. \end{aligned} \right. \quad (39)$$

The coordinates of points belonging to the characteristic curve are presented in Table 3, for a worm with dimensions: $m = 10$ mm; $R_b = \frac{m\cdot k}{2}\cdot\cos 20^\circ$; $R_e = \frac{m\cdot k}{2} + m$. The solution was found using the same algorithm as for square thread.

Table 3. Coordinates of points from the characteristic curve

X [mm]	Y [mm]	Z [mm]	X [mm]	Y [mm]	Z [mm]
58.100	-1.395	4.112	54.000	-1.244	3.393
58.000	-1.390	4.092	53.000	-1.217	3.252
57.000	-1.350	3.900	52.000	-1.195	3.130
56.000	-1.312	3.719	51.000	-1.182	3.030
55.000	-1.276	3.549	50.000	-1.185	2.966

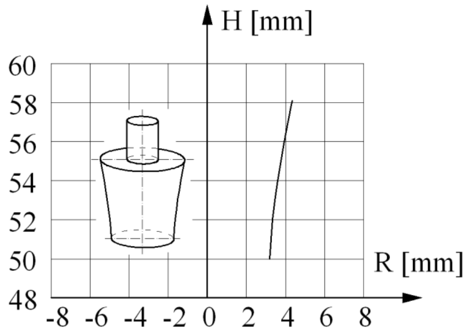

Fig. 10. Axial section of end mill tool

The form and coordinates of the axial section are presented in Figure 10 and Table 4.

Table 4. Coordinates of points from the axial section

R [mm]	H [mm]	R [mm]	H [mm]
58.100	4.342	54.000	3.614
58.000	4.322	53.000	3.472
57.000	4.127	52.000	3.350
56.000	3.944	51.000	3.252
55.000	3.771	50.000	3.194

4. CONCLUSIONS

The graphical method developed in CATIA using the program commands allows a rigorous description of the 3D model of helical surface. Also, the intersection curves of the helical surfaces with the orthogonal planes of the end mill axis and the relative generating trajectories can be rigorous described. The enveloping of generating trajectories family are parallel circles of the end mill tool.

The verification of the graphical solution with an analytical one can be made in the direct way using the coordinates of the characteristic curve, graphical determined, for calculation of the analytical condition (15).

In table 1, the "error" column represents the value of the equation (15) at points onto the characteristic curve. Theoretically, this value should be zero.

The determined errors are in domain of - $1\times10^{-3} \div 1.5\times10^{-2}$ which are small enough in order to compare the two methods.

5. ACKNOWLEDGEMENTS

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