GRAPHIC-ANALYTICAL STUDY OF TOOTH-LEVER DIFFERENTIAL GEAR MECHANISMS

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Abstract: The paper is devoted to graphic-analytical study of tooth-lever differential gear mechanism, used in roller technological machines with variable interaxial distance of working rollers, when the center of rotation of driven link performs rectilinear motion and lever link - complex plane-parallel motion. The aim of this work is to describe worked out method of investigation, to show the usefulness and benefits of this method. With worked out method as an example the studies of two types of tooth-lever differential gear mechanisms, principally different from each other, were carried out. The graphs of changes of gear relation of these mechanisms are given, depending on the changes of interaxial distance of leading and driven links. The method possesses visual simplicity, differs by convenience of results control and permits us promptly solve applied problems of design, as well as develops an engineering intuition to evaluate the possibilities of the mechanism according to its kinematic scheme.

Key words: graphic-analytical method, analysis, synthesis, tooth-lever mechanism, differential.

1. INTRODUCTION

The most perspective mechanisms to build modern machine and apparatus are tooth-lever mechanisms (Volmer, J., 1969; Levitsky, N., 1974). However the methods of analysis and synthesis of these mechanisms are poorly worked out and wait for their solution; this presents one of the most complex problems in this sphere. A special interest this problem provokes in connection with technical creativity, technical sciences, specific character of engineering thinking. The science of graphical building – descriptive geometry – gave those necessary scientific principles of geometrically graphical language for engineers, it became the bases of numerous graphical methods of solution of technical problems in XIX century. These methods were improved further in the first half of XX century and are being improved till now (Fedosova, S., 1984). A necessity of more accurate design of machines in real conditions of their operation facilitated the strengthening of analytical methods. At the same time graphic methods were worked out as well, in particular, it was connected with appearance of manipulators and robots; working organs of these devices perform complex spatial motions; requirements to such devices are higher. Success of analytical theory became possible thanks to preceding studies on geometric methods, which made a basis for general theory. Till now the most effective methods of analysis and synthesis of mechanisms were the methods, in which computational operations were combined with simultaneous viewing of geometrical characteristics of mechanisms by building, done by usual graphical ways or with the aid of displays and other automated devices to obtain graphic information from PC (Artobolevsky, I., 1977). So, it is indisputable, that graphical and graphical-analytical directions occupy considerable part in Machine Science. And still it stays the least studied one (Fedosova, S., 1984). As every mechanism, tooth-lever mechanisms may be studied by analytical, graphic-analytical and experimental methods. Formulae, derived by analytical way to calculate the parameters of tooth-lever differential mechanisms, are very bulky and demand an enormous amount of calculations, they do not possess visual simplicity; that was proved by a number of researchers (Levitsky, N., 1974; Frolov, G., et al., 2001; Fateyev, N., 2009; Karimov, R., 2012; and others). Graphic-analytical and graphical methods of study of differential mechanisms do not possess these shortcomings. The main shortcoming of these methods was comparative inaccuracy of results obtained. Development of a number of PC graphic programs, such as «AutoCAD», «SolidWorks», «Kompas-Grafik», «Mathcad» and others, eradicated this main shortcoming of the methods of graphic and graphic-analytical study, maintaining all its merits and gave a new impulse to the development of these methods. There are a number of examples, when graphic or graphic-analytical devices are the only acceptable ones, as they give the simplest solution of the problems. Besides, graphic-analytical and graphic methods of study due to their visual simplicity and convenience of control, are highly valuable for prompt check up of the correctness of analytical calculations, visual presentation of mechanisms.
The works of German researchers Mohr O., Prell R., Burmester L., Wittenauer P., Rittershau T., at al. (Mohr, O., 1875; Burmester, L., 1880; Wittenauer, P., 1923). The method of kinematic analysis of tooth gear mechanisms with the aid of velocity pattern was also offered by German researcher Carl Kutzbach (Kutzbach, C., 1925). This method is well presented in textbooks on the Theory of Machines and Mechanisms; it allows us to solve the problems of kinematic analysis of tooth mechanisms and is widely used in engineering and design of mechanisms. Recently published works on the study of mechanisms by graphic and graphic-analytical methods refer mainly to the study of differential and planetary mechanisms with rotational motion of the center of rotation of driven link along the circumference under constant interaxial distance between leading and driven tooth wheels. Thus Khlebosolov I.O. gives graphic methods of design of mechanisms with the aid of PC. He describes this method on an example of crank-slider mechanism, and as an instrument he uses drawing-graphic editing program «Kompas – Grafik», worked out by «ASCON» company. (Khlebosolov, I., 2004). Fateyev N.A. has offered graphic method, which allows to obtain visual pattern of angular velocities and angular accelerations of the links of tooth-lever mechanisms, given in Shashkin A.S. classification; these mechanisms present the ones with constant interaxial distance of driven and leading tooth wheels (Fateyev, N., 2009). Tretyakov V.M. notes that graphic and graphic-analytical methods are widely used in the study of the theory of mechanisms and machines not only due to their visual simplicity but due to the fact that they develop engineering intuition in students, which allows to evaluate kinematic possibilities of mechanism based on its kinematic scheme (Tretyakov, V., 2009). In his paper he has shown the use of “Mathcad” program at determination of velocities and accelerations of lever mechanisms. He has considered vector way of solution of a problem of kinematic analysis of mechanisms, based on the use of the method of vector polygons and theorems of velocities and accelerations of the points of rigid body. In design first the position of links is determined, and then characteristics of their rotational motion, further velocities and accelerations of the points (Tretyakov, V., 2011). This researcher has also shown graphic method of building of the pattern of distribution of angular velocities of tooth mechanisms. Here as an example are given: higher kinematic pairs of internal gearing, external gearing, cam mechanism, lever mechanism, planetary transmission of mixed type, bi-planetary transmission, closed planetary transmission and so on (Tretyakov, V., 2012). He notes that the use of offered method in addition to analytical one would allow students, who study tooth gears, to analyze in details the peculiarities of planetary mechanisms. From above mentioned it is seen that in literature those graphic methods of analysis of mechanisms are studied where lever link is subjected to rotational motion only; these are all types of differential and planetary mechanisms. Graphic method of analysis of tooth-lever mechanisms, where lever link performs complex plane-parallel motion is not described in literature. So the aim of this paper is to work out graphic-analytical method of study of tooth-lever differential gear mechanisms, applied in roller technological machines with variable interaxial distance of working rollers, where the center of rotation of driven link performs rectilinear motion and lever link – complex plane-parallel motion. Worked out method should allow to determine the position, linear and angular velocities, acceleration of characteristic points of the link as well as gear relation of a mechanism. This method meets all these requirements, as shown in this paper. Worked out method may be used in investigation of different types of tooth-lever differential gear mechanisms with variable interaxial distance of leading and driven links. The method possesses visual simplicity, differs in convenience of results control and allows promptly solve applied problems of projecting; it develops engineering intuition to evaluate the possibilities of a mechanisms based on its kinematic scheme. Design of new structures of differential tooth-lever gear mechanisms, development of graphic-dynamics and for synthesis of tooth-lever differential gear mechanisms, program design for automated investigation and projecting of these mechanisms with worked out methods will be an essence of our studies in the future.

2. METHOD OF GRAPHIC-ANALYTICAL STUDY OF KINEMATICS OF MECHANISMS

With worked out method as an example the study of two types of tooth-lever differential gear mechanisms, the first invented by us, (Abdulkarimov, A. at al. 1988) and the second offered by other authors (Kuznetsov, G. and Smirnov, B., 1967), was carried out (two types of mechanisms principally differ from each other). Linear and angular velocities of characteristic points of the links of mechanism were determined. As the most important criterion for tooth-lever differential gear mechanisms of such types is gear relation of mechanism and the character of its changes, the paper gives the graphs of changes
of gear relation of these mechanisms in dependence on the change of interaxial distance of driven and leading links.

2.1 Graphic-analytical method of kinematic study of tooth-lever differential gear mechanism with eccentric

This mechanism was suggested by B.I. Smirnov and is used in some roller machines (Kuznetsov, G. and Smirnov, B., 1967). Figure 1 shows graphical fulfillment of this method on the example of rack-lever differential gear mechanism with eccentric e Figure 1., a) shows the plan of gear mechanism, where: 1 - is a base box; 2, 3 - levers; 4 - slider; 5 - leading tooth wheel; 6, 7 - intermediate tooth wheel. From the condition of the problem we know the location of (ϕ, e) and geometrical parameters (l_{AB}, l_{BC}, l_{CD}, d_5, d_6, d_7, d_8) of this mechanism, and linear velocity of the center of rotation of driven tooth wheel in value and in direction (\vec{V}_D), angular velocity of leading tooth wheel 5 (\omega_5) also in value and in direction.

Taking the scale of mechanism (μ_i) we will draw the plan of mechanism in several positions, where (μ_i) is determined by an expression

\[ \mu_i = \frac{AB}{l_{AB}} \left[ \frac{mm}{m} \right] \]  

where AB - is a length of the link of mechanism in plan, [mm]; l_{AB} - true length of presented link in [m].

Instead of several positions of mechanism, to simplify graphic representation, it is desirable to draw several positions of lever contour of discussed mechanism. The plan of lever contour should be drawn also in scale (μ_i), corresponding to a scheme of mechanism (Fig. 1., b). Depending on the change of interaxial distance AD an angle ϕ is also changing. If interaxial distance is changing from AD_d to AD_0 where AD_0 - is minimal interaxial distance, and AD_d - maximal interaxial distance, an angle ϕ is changing from \phi_d to \phi_0 respectively (where \phi_d is maximal and \phi_0 minimal angles of crank position). So it is possible to divide the differences of angles in several equal positions (\phi_d - \phi_0) or the value of changes of interaxial distance (AD_d - AD_0). In our case the difference of angles is divided in 4 and 5 positions of mechanism are drawn over each angle \frac{\phi_d - \phi_0}{4}. On plotted plan of lever contour (Fig. 1., b) we will move characteristic points N and F from plan of mechanism into corresponding positions and we will have the points N_0, N_1, N_2, N_3, N_4, F_0, F_1, F_2, F_3, F_4.

Having built the plan of mechanism and plan of lever contour of this mechanism, we will draw the plan of velocities for each position (Fig. 1., c_1, c_2, c_3).

The velocity of a point D (\vec{V}_D) in plan of velocities is designated by segment \vec{Pd}, which is determined according to formula

\[ \vec{Pd} = \mu_i \cdot \vec{V}_D \]  

Where \mu_i \left[ \frac{mm}{ms^{-1}} \right] - is a scale of velocity.

To draw the plan of velocities we solve the system of equations

\[ \begin{align*}
\vec{V}_C &= \vec{V}_D + \vec{V}_{CD} \\
\vec{V}_C &= \vec{V}_B + \vec{V}_{CB}
\end{align*} \]  

hence, equating right sides of equation, we will have

\[ \vec{V}_D + \vec{V}_{CD} = \vec{V}_B + \vec{V}_{CB} \]  

Solving the equation (5) by graphic way, we determine the values of velocities \vec{V}_{CD} and \vec{V}_{CB}.

\[ \vec{V}_C = \vec{V}_{CB} = \frac{P_c}{\mu_i} \]  

\[ V_{CD} = \frac{dc}{\mu_i} \]

Solving lever contour of tooth-lever mechanism, we obtain linear and angular velocities of crank-slider four-link 12341.

Then we start the solution of tooth contour.

From the condition of the problem we know angular velocity of tooth wheel 5 (\omega_5) its separatory diameter d_5. Velocity of the point M is determined according to formula:

\[ V_M = \omega_5 d_5 \]
Fig. 1. Patterns of velocities of tooth-lever differential gear mechanism with eccentric e: a) plan of mechanism, 1 – base box, 2, 3 – levers, 4 – slider, 5, 6, 7, 8 – tooth wheels; b) plan of lever contour in different positions; c1), c2), c3) – plans of velocities of lever contour; d1), d2), d3) – patterns of velocities in I, II, III – positions of mechanism, respectively; e1), e2), e3) – patterns of angular velocities in I, II, III – positions of mechanism, respectively.
\[
\vec{V}_M = \omega_b \cdot r
\]  
(8)

Direction \(\vec{V}_M\) is known, \(\vec{V}_M \perp BA\).

Section of corresponding velocity \(\vec{V}_M\) is determined by formula

\[
\vec{M}_iM'_i = \vec{V}_M \cdot \mu_v
\]  
(9)

Drawing vertical line \(I-I\) (for the first position of mechanism) parallel to the line, crossing the axes of leading and driven tooth wheels \((AD)\), we will move to this line by horizontal rays the points \(A, M, D, B, N, C, F\) (Fig. 1., \(d_i\)), obtained points are marked as \(A_1, M_1, B_1, N_1, C_1, F_1, D_1\) respectively.

The segment \(\overline{M_iM'_i}\) is moved on point \(M_i\) (Fig. 1., \(d_i\)), we project it on horizontal line and obtain the segment \(\overline{M_iM''_i}\). From the point \(M''_i\), we will line inclined ray, crossing the point \(A_i\) and designate it by numeral 5. This ray will be the line of distribution of velocities of wheel 5.

An angle of ray 5 with line \(I-I\) gives angular velocity of wheel 5 \((\omega_5)\).

It should be noted, that in projecting of the vectors of velocities on horizontal line and their perpendiculars (that is radius-vectors of these velocities) – on vertical line, their relations, that is, angular velocities, remain invariable in value.

\[
\omega_5 = \frac{V_M}{l_{MA}} = \frac{M_iM'_i \cdot \mu_l}{\mu_v \cdot AM} = \frac{\mu_l \cdot M_iM''_i}{\mu_v \cdot A_1M_i} \cdot \cos \alpha
\]

\[
= \frac{\mu_l \cdot M_iM''_i}{\mu_v \cdot A_1M_i} = \frac{\mu_l \cdot M_iM''_i}{\mu_v} \cdot \frac{A_1M_i}{A_1M_i}
\]  
(10)

Therefore,

\[
\omega_5 = \frac{\mu_l \cdot M_iM''_i}{\mu_v}
\]  
(11)

Ray crossing the points \(M''_i\) and \(B_i\) is a line of distribution of velocities of tooth wheel 6. Running horizontal line from point \(N_i\) of the plan of lever contour (Fig. 1., \(b\)) till the crossing with ray 6 in the plan of a pattern of velocities of position \(I\) (Fig. 1., \(d_i\)) we will get the segment \(\overline{N''_iN_i}\) (Fig. 1., \(b\)), which is a projection of the vector of velocities \(V_N\) on horizontal line.

To the point \(C_i\) (Fig. 1., \(b\)) we will move the vector of velocity of point \(C\) \((V_C)\) in plan of velocities of lever contour for the first position, this is the segment \(\overline{C_iC''_i}\) from the plan of velocities. Obtained segment \(\overline{C_iC''_i}\) is projected on horizontal line, and we get the segment \(\overline{C_iC''_i}\), Segment \(\overline{C_iC''_i}\) is moved on point \(C_i\) in vertical line \(I-I\) (Fig. 1., \(d_i\)).

Connecting found points \(C''_i\) and \(N_i''\) we get ray 7, which is the ray of distribution of velocities of a wheel 7, and crossing point of ray 7 with vertical line \(I-I\) is the point of instantaneous center of rotation \((P_7)\) of wheel 7 in plan of a pattern of velocities.

Connecting point \(C''_i\) with point \(B_i\), we get the line of distribution of velocities of crank 2 (2) (Fig. 1., \(d_i\)).

From the point \(F_i\) (Fig. 1., \(b\)) we will run horizontal line till the crossing of vertical line \(I-I\) and will get point \(F_i\) in plan of a pattern of velocities of position \(I\) (Fig. 1., \(d_i\)) and further till crossing the ray 7, we will get the point \(F''_i\) (Fig. 1., \(d_i\)).

A vector \(\overline{F_iF''_i}\) is a picture of the linear velocity of the contact point \(F_i\).

Running the lines from points \(C''_i\) and \(F''_i\), crossing the points \(D_i\), we will get the lines of distribution of velocities 3 and 8 for lever 3 and tooth wheel 8. Obtained angles \(\psi_2, \psi_3, \psi_5, \psi_6, \psi_7, \psi_8\) between vertical line \(I-I\) and rays of distribution of velocities are the patterns of angular velocities in plan. Dividing angles in scale of angular velocities, we get actual angular velocities of links

\[
\omega = \frac{\psi}{\mu_\circ}
\]  
(12)

where,

\[
\mu_\circ = \frac{\psi_5}{\omega_5}
\]  
(13)

To get a visual pattern of angular velocities and frequencies of rotation of tooth wheel we will select a common point \(O_i\) (Fig. 1., \(e_i\)), through which runs a beam of rays parallel to corresponding lines of distribution of velocities, that is rays with angles of incline \(\psi_2, \psi_3, \psi_5, \psi_6, \psi_7, \psi_8\). If this beam of rays is crossed by some line, perpendicular to vertical line \(I-I\) of the line of counting of linear velocities, the points of cross-section 2, 3, 5, 6, 7, 8 are noted and segments \(O2, O3, O5, O6, O7, O8\) are obtained, counted from the origin point of count out \(O\).

Dividing these segments in scale of angular velocities, which includes segment \(OO_1\), we may determine true angular velocities of links; here the
scale of angular velocities is determined by formula

$$\mu_\omega = \frac{\mu_l}{\mu_i} \frac{OO_l}{\hat{r}_{mm}} \cdot \text{deg} \cdot \text{s}^{-1}$$  \hspace{1cm} (14)$$

Having in mind, that \( n = 2\pi \omega \), we may determine the scale of the frequency of rotation

$$\mu_n = 2\pi \mu_\omega$$  \hspace{1cm} (15)$$

Gear relations of mechanism (of wheels 5 and 8) are determined by relationship

$$u = \frac{\omega_1}{\omega_3} = \frac{OO_8}{OO_5}$$  \hspace{1cm} (16)$$

By the same way we will build plans for other positions of mechanism (Fig. 1., \( c_2, c_3, d_2, d_3, e_2, e_3 \).)

While building the next positions one should pay an attention to the fact that the points of contact of tooth wheels (points \( N_0, N_1, N_2, N_3, N_4, F_0, F_1, F_2, F_3, F_4 \) change their position along vertical axis (all but contact point \( M \)).

Obtaining segments, proportional to angular velocities of levers (\( O2, O3 \) and tooth wheels (\( O5, O6, O7, O8 \)) we will build the graph of changes of angular velocities of levers and tooth wheels, depending on the change of interaxial distance of driven and leading tooth wheels.

In Fig. 2 graphs of angular velocities of tooth wheels 5, 6, 7, 8 (\( \omega_5, \omega_6, \omega_7, \omega_8 \)), levers 2, 3 (\( \omega_2, \omega_3 \)) and transmission ratio of mechanism (\( u \)) are shown at constant velocity of leading tooth wheel \(( \omega_3 \) and variable linear velocity of the center of rotation of driven tooth wheel \( \bar{V}_D \)).

![Graph of angular velocities](image)

Fig. 2. Graphs of angular velocities \( \omega_2, \omega_3, \omega_5, \omega_6, \omega_7, \omega_8 \) and transmission ratio of mechanism \((u)\) in dependence of velocity of the center of rotation of driven link \( \bar{V}_D \)

Analysis of graphs shows that with the presence of linear velocity \( \bar{V}_D \) of the center of rotation of driven tooth wheel \( \delta \), even at constant angular velocity of leading tooth wheel \(( \omega_1 \), an angular velocity of this wheel \( \omega_1 \) will be variable; that leads to variability of gear relation of discussed gear mechanism. In this case angular velocity of tooth wheel 6 \(( \omega_6 \) also remains constant, and angular velocities of tooth wheel 7 \(( \omega_7 \) levers 2 and 3 \(( \omega_2, \omega_3 \) are variables. The difference of angular velocities \( \omega_5, \omega_6, \omega_8 \) and \( \omega_2, \omega_3 \) from \( \omega_5 \) depends on the value and direction of velocity of the center of rotation of driven link \( \bar{V}_D \).

2.2 Graphic-analytical study of kinematics of tooth-lever differential gear mechanism with internal and external gearing

In Fig. 3 graphic realization of kinematics of worked out by the authors tooth-lever differential gear mechanism with external and internal gearing (Abdukarimov, A., 1988) is shown by graphic-analytical method. Fig.1., shows a) plan of gear mechanism, where: 1 – is base box; 2, 3 – levers; 4 – slider; 5 – leading tooth wheel; 6 – intermediate tooth wheel; 7 – driven tooth wheel.

Position \(( \varphi \) ), geometric (\( l_{AB}, l_{BC}, d_5, d_6, d_8 \)) and kinematic \(( \omega_2, \bar{V}_C \) parameters of the mechanism are known.

For a change, besides the mechanism, shown in Fig. 1., consider the case, when the center of rotation of driven tooth wheel is directed to the center of rotation of leading tooth wheel with velocity \( \bar{V}_C \). Instead of angle \( \varphi \) we will take an angle \( \alpha \), since both angles are the functions of the way from velocity \( \bar{V}_C \). Assuming angle \( \alpha \) within the limits from 0° to 180° and dividing it in every 45°, we will draw lever contour in five positions (Fig. 1., b).

We will note in plan of lever contour all characteristic points of mechanism \( (M, M', B, C, N) \) in five positions and will obtain the points \( M'_1, ..., M'_5, M'_7, ..., M'_8, B'_1, ..., B'_8, N'_1, ..., N'_8 \). Further proceeding with graphic-analytical solution as in 2.1 we determine linear and angular velocities of characteristic points in all positions of mechanism.

As the most important for us is gear relation \((u)\) of mechanism, we determine it in all positions according to formula:

$$u_{1/5} = \frac{\omega_5}{\omega_3}$$  \hspace{1cm} (17)$$

Fig. 4 shows the graphs of angular velocities of tooth wheels 5, 6, 7 \(( \omega_5, \omega_6, \omega_7 \) and levers 2, 3 \(( \omega_2, \omega_3 \) at constant angular velocity of leading tooth wheel
Fig. 3. Patterns of velocities of tooth-lever differential gear mechanism with external and internal gearing: 

- **a)** plan of mechanism, 1 – base box, 2, 3 – levers, 4 – slider, 5, 6, 7, 8 – tooth wheels;
- **b)** plan of lever contour in different positions; 
- **c)** – plans of velocities in II, III, IV – positions of mechanism, respectively;
- **d)** – pattern of velocities in II, III, IV – positions of mechanism, respectively;
- **e)** patterns of angular velocities in II, III, IV – positions of mechanism, respectively.
velocities $\omega_3$ and variable linear velocity of the center of rotation of driven tooth wheel ($\bar{V}_C$).

Analysis of graphs show that with the presence of linear velocity $V_C$ of the center of rotation of driven tooth wheel ($7$), even at constant $\omega_3$, angular velocity $\omega_5$ will be variable, that leads to variability of transmission ratio of mechanism ($u_{1/3}$). Angular velocities $\omega_2$, $\omega_1$, $\omega_6$ are also variable. The difference of $\omega_7$ from $\omega_5$ depends on value and direction of velocity of the center of rotation of driven link ($\bar{V}_C$).

3. CONCLUSIONS

Graphic-analytical method of kinematic study of tooth-lever differential gear mechanisms is worked out. It is shown, that in discussed gear mechanisms, at the change of interaxial distance of leading and driven links, their relation will be variable.

An increase or decrease of gear relation from initial value depends on direction and the value of linear velocity of the center of rotation of driven link.

4. REFERENCES


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