

# EXPERIMENTAL STUDIES AND COMPUTER SIMULATION OF STRESS IN PLATES WITH ROUND CORNERS CUTTINGS

Anișoara-Gabriela Cristea, Costel Iulian Mocanu

“Dunarea de Jos” University of Galati, Faculty of Naval Architecture, Stiintei Street, No. 2, Galati, 800146, Romania

Corresponding author: Anișoara-Gabriela Cristea, [anisoara.cristea@ugal.ro](mailto:anisoara.cristea@ugal.ro)

**Abstract:** This paper presents both experimental verifications and numerical modeling regarding the determination of the induced stresses and of the specific deformations on a plexiglass structure provided with a cutting. Calculation by analytical methods of plates having a certain configuration and presenting some cuttings is difficult to achieve, in such cases appealing to various numerical methods, the most commonly used being the finite element method (FEM). The idea of FEM consists in modeling the field studied through a set of finite-size elements, connected to each other in a number of nodal points in which solution is searched. Errors between experimental results and those obtained numerically can be also generated by some objective factors: the structure used in the experiment is not new, being likely to be deformed, the conditions required can make the structure rigid enough and also reading error of displacements can exist. Experimental methods represent the means by which analytical or numerical calculations can be verified. The numerical model presented, as well as some assumptions that were adopted in its achievement, required an experimental verification for its validation, as well as increasing confidence in the methods used. Following the results obtained by experimental and numerical means, a deviation of 5.3% was found which proves that it falls within the allowable limits (10%).

Experimental measurements were made on a reduced scale physical model of the real structure, allowing quantitative verification of the results obtained by calculation and evaluation of their accuracy. Local character sizes such as maximum Von Mises stresses were checked.

**Key words:** specific deformations, normal stresses, numerical modeling.

## 1. INTRODUCTION

The general methodology of plates analysis assumes writing equations of equilibrium for the plate element considered (Domnisoru L., 2009): writing relationships between displacements and specific deformations, also named geometric compatibility relations, which represent the geometric aspect of the problem; writing the relationship between stresses and specific deformations which is the physical aspect of the problem.

The idea of FEM consists in modeling the field studied through a set of finite-size elements, connected to each other in a number of nodal points in which solution is

searched.

At plates in plane state of stress, in nodal points of finite elements generally the displacements  $u$  and  $v$  are determined – which are called generalized displacements or degrees of freedom.

The connection between elements is done in the assembly process, which consists in writing equations of equilibrium of the nodes where given external forces are applied and forces of interaction between elements – called generalized forces. The relationship between generalized displacements and generalized forces of an element is made through the stiffness matrix, usually obtained on the Ritz or Galerkin methods.

The usual presentations of FEM primarily aim at the following steps:

- introduction of general data regarding geometry, the material and structure interactions with the outside – loads and links; discretization in finite elements;
- obtaining the stiffness matrices and vectors of small loads to nodes based on generally accepted shape functions for the displacements field;
- assembly of the elements, implementation of boundary conditions and solving the system of equations;
- determination of displacements and the state of stress and strain in each finite element.

## 2. MEASUREMENT OF DEFORMATIONS AND OF ELONGATIONS OF A SOLID BODY

### 2.1 Method of measurement

Electrical tensometry is the method of measurement of deformations and of elongations of a solid body, through transducers that convert mechanical deformation variations in variations of an electrical quantity.

As method, electrical tensometry is part of the general methods of electrical measurement of non-electric quantities.

### 2.2 The device used to measure deformations and elongations

- a) Resistive transducer

Resistive transducer used in tensometry is a resistor built from one or more metallic conductors connected in series, with a very small diameter (0.015...0.02 mm), having a wasteful resistance whose values are usually between  $R = 50\Omega$  and  $R = 1000\Omega$ .

Due to its small shape and size (figure 1) the resistive transducer is also named strain gauge or tensometric stamp (William N. Sharpe &Jr., William N. Sharpe 2008 ).

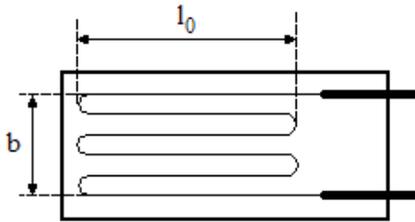


Fig. 1. Resistive transducer

b) Principle of operation of the resistive transducer

The transducer is bonded to the piece subject to research in order to pursue its deformations. This transducer suffers by deformation a variation of its wasteful resistance. It was found that the specific variation of the transducer resistance is actually, within certain limits, proportional to the specific deformation suffered by it along with the piece on which it is applied.

**3. NUMERICAL MODELINGS. EXPERIMENTAL VERIFICATIONS**

The model for finite element analysis has to synthesize efficiently all available information relevant to that structure.

Both numerical and experimental studies were conducted on a plexiglass structure (Cristea, 2009).

Numerical modeling as well as experimental verifications was conducted on a plexiglass structure, provided with a cutting with the following general characteristics (table 1):

Table 1. General characteristics of the plexuglass plate

Length of the plate L [mm]	420
Height of the plate B [mm]	240
Thickness of the plate [mm]	8
Length of cutting l [mm]	180
Height of cutting b [mm]	120
Radius of cutting r [mm]	30, 20, 10

For modeling and analysis of the structure, the system of analysis programs finite element COSMOS / M version 2.6 was used (x x x – COSMOS/M 2.6 User’s Guide, Structural Research and Analysis Corporation (SRAC), CA-Los Angeles, 1999-2000).

A plate provided with a cutting subject to stresses (figure 2) was considered.

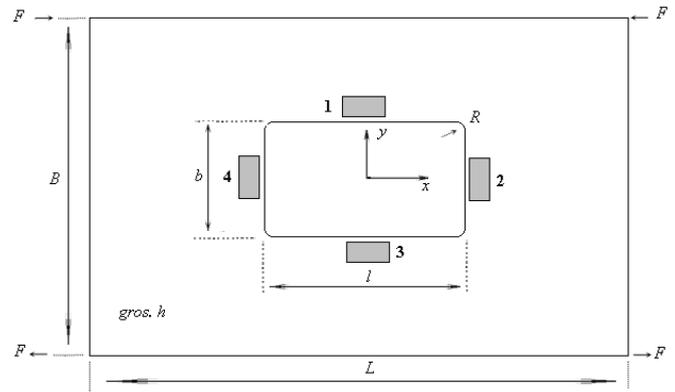


Fig. 2. Plate loaded with concentrated forces

It was considered the stress presented due to the fact that for conducting the experiments it was relatively difficult to simulate a real load with distributed loads. For numerical performance of load with concentrated forces the idea of using on the small sides of the model of two rigid bars was adopted.

For modeling the plate, structure triangular elements SHELL3T type and for the two bars BEAM3D type bar elements were used.

Table 2 presents the characteristics of CAD-FEM model and the characteristics of the material for the two structures.

Table 2. Characteristics of CAD - FEM

Number of triangular elements, SHELL3T	1674
Young’s modulus, E [N/mm <sup>2</sup> ]	2,1E+11
Poisson’s ratio, $\nu$	0,403
Number of bar elements, BEAM3D	98
Diameter of the bar, $\Phi$ [mm]	1000

Loading conditions are those used in experimental tests, force is concentrated at the ends of the two bars with high stiffness.

Model geometry and edge conditions, as well as external loading are presented in figure 3.

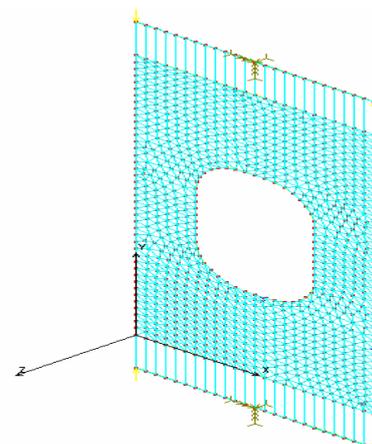


Fig. 3. Model geometry and edge conditions and external loading

The diagram of the resulting stresses in the directions of interest in the case of maximum force of application is presented in figure 4 and figure 5.

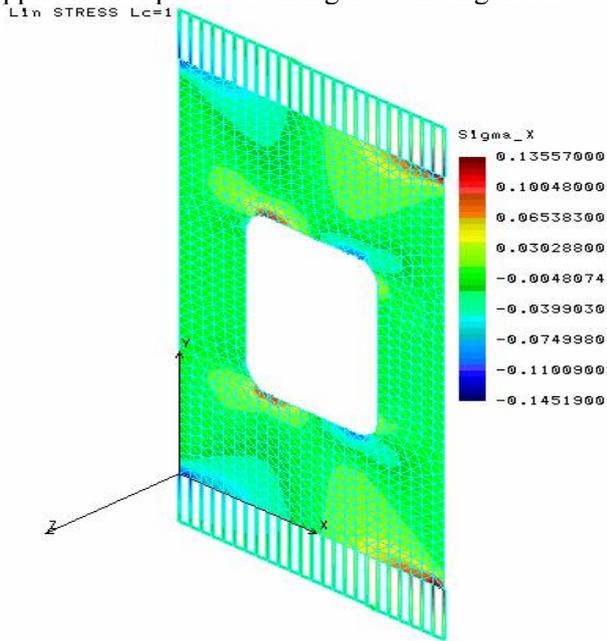


Fig. 4. Normal stresses in plate  $\sigma_x$

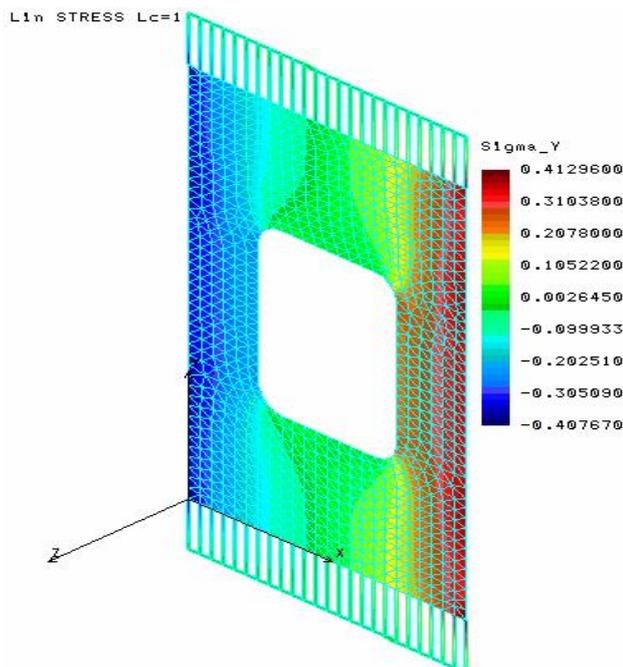


Fig. 5. Normal stresses in plate  $\sigma_y$

### 3.1 Experimental verifications

For the present research approach it was chosen a structure provided with a relief cutting of a framing element, to determine the maximum stresses and specific deformations and to compare them with admissible values.

The experiments were carried out in the laboratory of the Department of Strength of Materials and the aims were to compare theoretical and practical results and to determine errors to validate numerical modeling.

*Model and experimental stand*

The experimental stand is composed of two supports (upper and lower) made by joining two angles with equal wings 60x60x6 mm, four fixing elements made of platband with the following dimensions 250x30x4 mm, two tensioning elements made of two threaded rods at the ends left - right and four studs threaded right - left with normal pace. Fixing elements, as well as the studs are welded on the two supports.

The plexiglass plate is mounted between the two supports (upper and lower) using fixing elements. The strain gauges were placed in the direction of stress very close to the contour of cutting.

### 3.2 Experimental results

The experiment was conducted for a range of cutting area connection of  $r = 30$  [mm],  $r = 20$  [mm] and  $r = 10$  [mm]. The experimental model is presented in figure 6.



Fig. 6. Experimental model

The experimental methodology used is electrical resistive tensometry.

The experimental results obtained are provided by the strain gauges numbered 1, 2, 3, 4 placed on the steel model (figure 7).

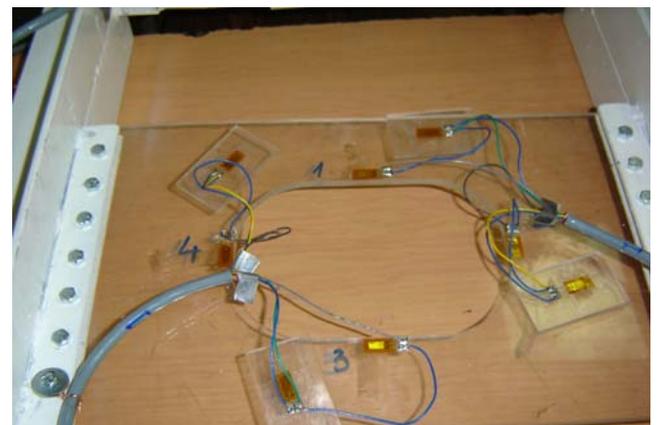


Fig. 7. Arrangement of strain gauges on x and y

Specific deformations measured, as well as normal stresses measured are presented in tabular form.

a) Variations in specific deformations and normal stresses depending on the force applied are presented in table 3 and 4 in case the connection radius of the cutting area is 30 mm.

Table 3. Specific deformations

Force [daN]		0,0	4,7	5,7	7,1	8,6
$\epsilon_{\text{read}}$ [ $\mu\text{m}/\text{m}$ ]	M <sub>1</sub>	-0,01	-75,16	-86,78	-117,21	-135,78
	M <sub>2</sub>	0,00	-0,79	-0,92	-1,01	-1,32
	M <sub>3</sub>	0,00	72,44	85,33	116,38	136,09
	M <sub>4</sub>	-0,01	-0,31	-0,39	-3,49	-0,76

Table 4. Normal stresses resulted from the numerical modeling and experiment

Measuring point	Force [N]	Mark 1	Mark 2	Mark 3	Mark 4
		[Pa]			
Numerical modeling	47	-2,418E+05	-2,001E+03	2,406E+05	-1,512E+03
Experiment		-2,356E+05	-2,176E+03	2,271E+05	-1,688E+03
Numerical modeling	57	-2,933E+05	-2,427E+03	2,918E+05	-1,834E+03
Experiment		-2,720E+05	-2,584E+03	2,675E+05	-1,722E+03
Numerical modeling	71	-3,653E+05	-3,024E+03	3,635E+05	-2,285E+03
Experiment		-3,614E+05	-3,166E+03	3,548E+05	-2,094E+04
Numerical modeling	86	-4,425E+05	-3,663E+03	4,403E+05	-2,768E+03
Experiment		-4,256E+05	-4,038E+03	4,266E+05	-2,582E+03

Comparative graphs regarding the variation mode of the normal stresses for each separate mark (figures 8, 9, 10, 11).

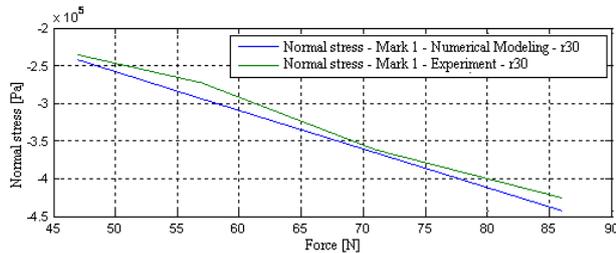


Fig. 8. Variation of normal stresses – Mark 1

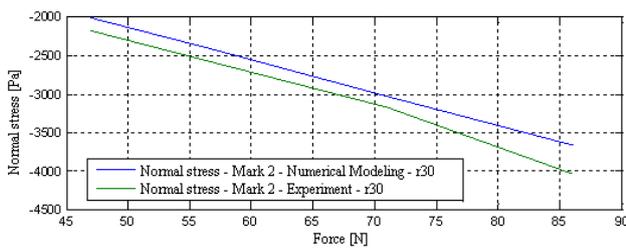


Fig. 9. Variation of normal stresses – Mark 2

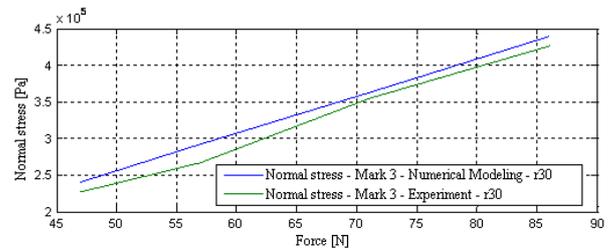


Fig. 10. Variation of normal stresses – Mark 3

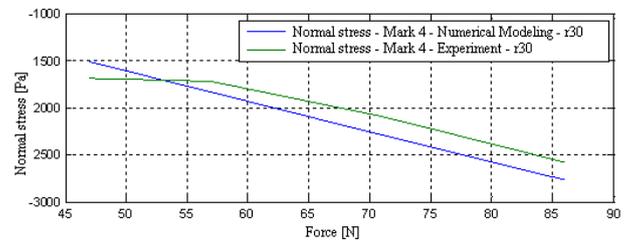


Fig. 11. Variation of normal stresses – Mark 4

b) Variations in specific deformations and normal stresses depending on the force applied are presented in table 5 and 6 in case the connection radius of the cutting area is 20 mm.

Table 5. Specific deformations

Force [daN]		0,0	4,7	5,7	7,1	8,6
$\epsilon_{\text{read}}$ [ $\mu\text{m}/\text{m}$ ]	M <sub>1</sub>	-0,01	-73,86	-85,12	-125,17	-133,45
	M <sub>2</sub>	0,01	-0,73	-0,89	-0,98	-1,28
	M <sub>3</sub>	0,00	73,23	85,56	114,78	134,79
	M <sub>4</sub>	0,01	-0,29	-0,37	-0,39	-0,70

Table 6. Normal stresses resulted from the numerical modeling and experiment

Measuring point	Force [N]	Mark 1	Mark 2	Mark 3	Mark 4
		[Pa]			
Numerical modeling	47	-2,366E+05	-2,145E+03	2,351E+05	-1,375E+03
Experiment		-2,315E+05	-2,288E+03	2,295E+05	-1,291E+03
Numerical modeling	57	-2,870E+05	-1,389E+03	2,852E+05	-1,668E+03
Experiment		-2,668E+05	-1,490E+03	2,682E+05	-1,522E+03
Numerical modeling	71	-3,574E+05	-3,730E+03	3,552E+05	-2,078E+03
Experiment		-3,924E+05	-3,672E+03	3,598E+05	-1,894E+04
Numerical modeling	86	-4,330E+05	-3,896E+03	4,035E+05	-2,517E+03
Experiment		-4,183E+05	-4,012E+03	4,225E+05	-2,382E+03

Comparative graphs regarding the variation mode of the normal stresses for each separate mark (figures 12, 13, 14, 15).

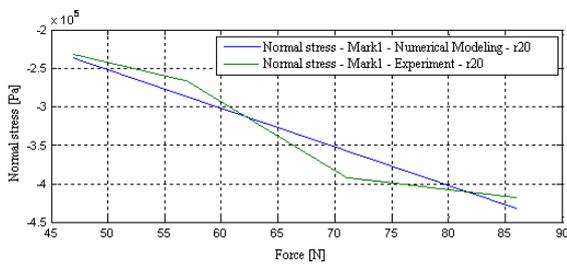


Fig. 12. Variation of normal stresses – Mark 1

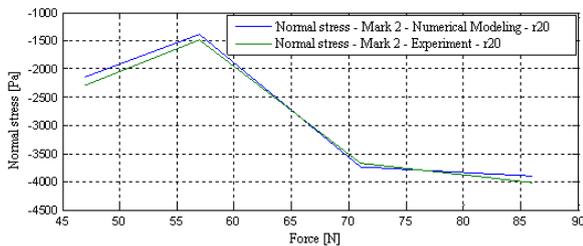


Fig. 13. Variation of normal stresses – Mark 2

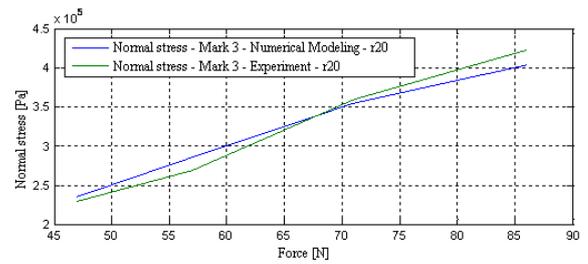


Fig. 14. Variation of normal stresses – Mark 3

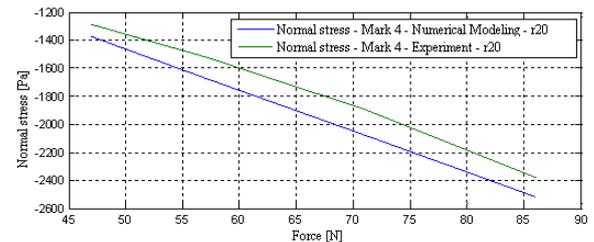


Fig. 15. Variation of normal stresses – Mark 4

c) Variations in specific deformations and normal stresses depending on the force applied are presented in table 7 and 8 in case the connection radius of the cutting area is 10 mm.

Table 7. Specific deformations

Force [daN]	0,0	4,7	5,7	7,1	8,6	
$\epsilon_{read}$ [ $\mu\text{m}/\text{m}$ ]	M <sub>1</sub>	0,01	-72,16	-83,33	-123,54	-129,56
	M <sub>2</sub>	-0,02	-0,70	-0,85	-0,96	-1,24
	M <sub>3</sub>	0,00	71,89	83,27	124,57	130,26
	M <sub>4</sub>	0,01	-0,25	-0,33	-0,36	-0,68

Table 8. Normal stresses resulted from the numerical modeling and experiment

Measuring point	Force [N]	Mark 1	Mark 2	Mark 3	Mark 4
		[Pa]			
Numerical modeling	47	-2,323E+05	-2,326E+03	2,307E+05	-1,497E+03
Experiment		-2,262E+05	-2,194E+03	2,253E+05	-1,637E+03
Numerical modeling	57	-2,817E+05	-2,821E+03	2,798E+05	-1,816E+03
Experiment		-2,612E+05	-2,664E+03	2,610E+05	-1,634E+03
Numerical modeling	71	-3,509E+05	-3,513E+03	3,486E+05	-2,262E+03
Experiment		-3,873E+05	-3,409E+03	3,605E+05	-2,128E+03
Numerical modeling	86	-4,250E+05	-4,256E+03	4,222E+05	-2,740E+03
Experiment		-4,061E+05	-3,887E+03	4,083E+05	-2,531E+03

Comparative graphs regarding the variation mode of the normal stresses for each separate mark (figures 16, 17, 18, 19).

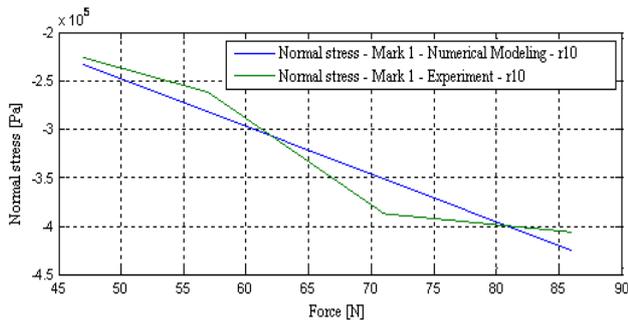


Fig. 16. Variation of normal stresses – Mark 1

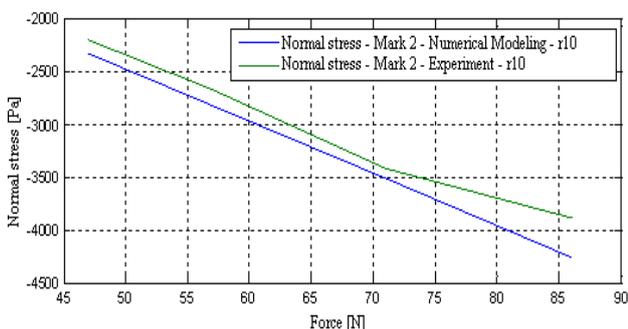


Fig. 17. Variation of normal stresses – Mark 2

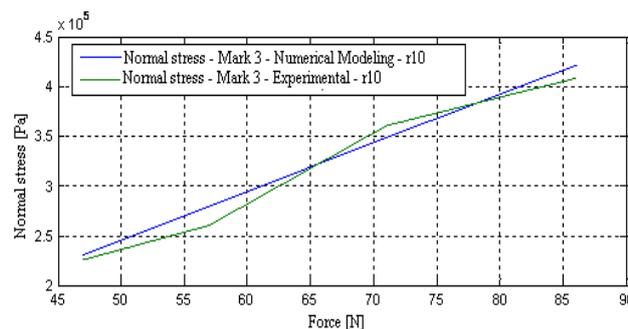


Fig. 18. Variation of normal stresses – Mark 3

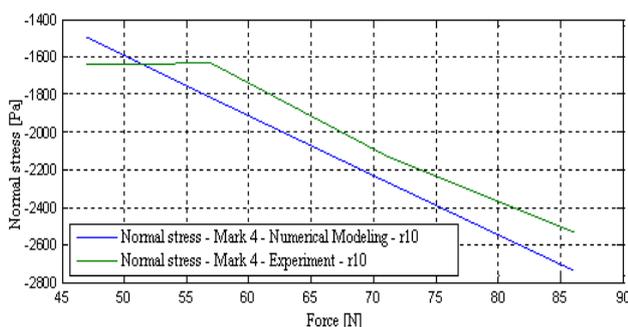


Fig. 19. Variation of normal stresses – Mark 4

#### 4. CONCLUSIONS

The general methodology of plates analysis assumes writing equations of equilibrium for the plate element considered; writing relationships between displacements and specific deformations, also named

geometric compatibility relations, which represent the geometric aspect of the problem; writing the relationship between stresses and specific deformations which is the physical aspect of the problem.

Following the results obtained by experimental and numerical means, a deviation of 5.3% was found which proves that it falls within the allowable limits (10%).

Given the deviation above presented it can be considered that numerical modeling is well verified experimentally and can still be used for the study of resistance elements that goes into the hull whose shape is similar to that presented.

Also, it should be taken into account that the FEM is approximate, which means that the model cannot be asked more than the method can provide, the results obtained being determined both by the performances of the model, and by the principles, assumptions and mathematical procedures of calculation included in the method and program with finite elements. The problem of approximation errors of modelling and finite element analysis is very complex, making it almost impossible to control and evaluate them.

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