DESIGN METHODOLOGY OF KINETIC HYDRAULIC TURBINES

Eugen-Vlad Năstase, Dan Scurtu, Doru Călăraşu & Bogdan Ciobanu

"Gheorghe Asachi" Technical University of Iasi-Romania, Department of Fluid Mechanics
Blvd. Mangeron No. 67, 700050 Iasi, Romania

Corresponding author : Eugen-Vlad Năstase, nastase_eugenvlad@yahoo.com

Abstract : The main energy that uses this type of turbine is the kinetic energy of water. The rotor is the principal element of turbine. The rotor is a propeller that captures a moving fluid energy. In this type of propeller, the fluid flow moves around it so that the forces are the result of direct action of the fluid on the blades. Normal force is an axial force, to be taken over by the supporting structure. Tangential component is the sense of an active force that produces torque on the propeller shaft.
This component tends to increase the speed of the propeller, the propeller being able thus be used as a power source. In other words turbine function according to this principle is a power plant.
Propeller calculation aims to determine the diameter of the propeller and propeller shaft torque. Propeller diameter is determined taking into account the imposed power and fluid flow speed and the torque is determined by the action of fluid flow on the propeller blades.
Therefore the type of profile chosen for the construction of aero-hydrodynamic blade will decisively influence the performance of the propeller.
Key words: water turbine, rotor, blade, momentum theory

1. INTRODUCTION

This type of turbine interacts with the water, capturing part of its kinetic energy and converting it into usable energy. The present energetic context leads to the research of renewable energy sources and the recuperation of kinetics energy from water currents is particularly interesting (Kulunk&Yilmaz, 2009). Although current research on the possibilities to capture the kinetic energy of water is not published design methodology for such turbines. Design methodology used in this work is the same as that of wind turbines. The principal element of turbine is the rotor, which is a propeller that captures a moving fluid energy. The forces for this type of propellers are due to direct fluid flow over the blades. Resultant force of this interaction can be decomposed into a normal and a tangential component. Normal force is an axial force, pushing the positive, to be taken by the supporting structure. Tangential component is an active force that produces torque on the propeller shaft. This component tends to increase the rotational speed of the propeller so that the propeller can be used as a power source. Calculation propeller has two goals :

- determination of the propeller diameter D. This size is determined from imposed power and speed of fluid flow.
- calculating torque from the propeller shaft. Torque is determined by the action of fluid flow on the propeller blades. From this point of view the hydrodynamic profile type used for blades construction, will decisively influence the propeller performance.

2. GENERAL INFORMATION

Design method allows the establishment of the propeller geometry and hydrodynamic performance was found after a profile that will be used in blade construction. The method aims at establishing the turbine shaft torque and power coefficient. Axial rotor theory is not unitary, it appears that a mix of theoretical models and variants that overlap in some areas. The difference between models is generated by assumptions simplifying and approximations from which the general three-dimensional movement can be replaced by a simple motion, uniquely characterizes the essence of core theory. Refer to a series of approximations, namely :
-Models with infinite number of blades, by neglecting the periodicity leads to an axially symmetric flow. Models with infinite number of blades are specific for free propeller where free end of the blade determine the three-dimensional effects with negligible intensity ;
-The ideal propeller, characterized by axially symmetric motion without taking into account the tangential velocities ;
-The imaginary rotor characterized by axially symmetric motion considering and the tangential velocities. It is found that neglecting the periodicity of the motion is to replace real rotor blades with an infinite number of rotor blades having a finite number of negligible thicknesses. Generated surface will confuse the rotor. When crossing the area, both velocities and pressures will show how discontinuities.

Rotor with a finite number of blades can be treated with an active disk. From this point of view, active disk model will comprise the model of ideal propeller and the model of imaginary rotor. The propeller is an active disk that captures the kinetic energy part of the current. It is placed in a stream of fluid whose flow velocity at infinity upstream is $V_\infty = V_\infty\, i_1$.

3. MATHEMATICAL MODEL

From the above results that the motion around an ideal propeller can be characterized as follows:

- axially symmetric motion is no tangential speed;
- velocities have the form of absolute speeds are scaling the components :

$$v_1(x)=u_\infty+u_1(x) \quad v_2(x)=u_2(x) \quad v_3(x)=u_3(x) \qquad (1)$$

- pressure variation across the active disk is the same at any point and velocity is continuous.

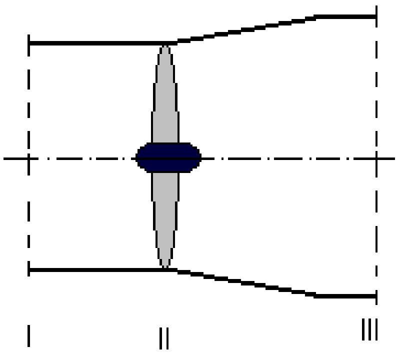

Fig.1 Geometry of a stream tube

Start from the equations of continuity and momentum we have relations :

$$\int_D \rho < \vec{v}, \vec{n} > da = 0$$
$$\int_D \rho\vec{v} < \vec{v}, \vec{n} > da = \int_D \rho\vec{f} dv - \int_D P\vec{n} da + F \qquad (2)$$

Consider an elementary subdomain determined by two current neighboring areas d=d-Ud₊ , as can be seen in figure 2.

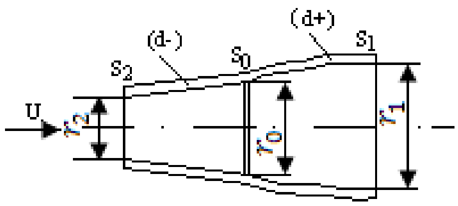

Fig.2. Subdomain work item

From the expression of the continuity equation we have:

$$\rho u_\infty * 2\pi r_1 dr_1 = \rho(u_\infty + u_1(0, r_0))2\pi r_0 dr_0 = \rho(u_\infty + u_1(\infty, r_2))2\pi r_2 dr_2 \qquad (3)$$

The momentum equation applied to (d-) şi (d+) resulting in projection after $0x_1$:

$$-\rho u_\infty^2 2\pi r_1 dr_1 + \rho(u_\infty + u_1(0, r_0))^2 2\pi r_0 dr_0 = 2\pi r_1 dr_1 P_\infty - P(-0, r_0)2\pi r_0 dr_0 \qquad (4)$$

$$-\rho(u_\infty + u_1(0, r_0))^2 2\pi r_0 dr_0 + \rho(u_\infty + u_1(\infty, r_2))^2 * 2\pi r_2 dr_2 = P(+0, r_0)2\pi r_0 dr_0 - P_2(r_2) * 2\pi r_2 dr_2 \qquad (4`)$$

With previous relationships we can determine the axial force (5) for a elementary rotor which was obtained by intersection between (d) domain and the disc (S_0):

$$dT(r_0) = (P(+0, r_0) - P(-0, r_0)) * 2\pi r_0 dr_0 = \rho(u_\infty + u_1(0, r_0))u_1(\infty, r_2) * 2\pi r_0 dr_0 + P_\infty 2\pi r_1 dr_1 - P_2(r_2)2\pi r_2 dr_2 \qquad (5)$$

From Bernoulli`s equation:

$$\frac{1}{2}P_\infty + \frac{1}{2}u_\infty^2 = \frac{1}{\rho}P(-0, r_0) + \frac{1}{2}(u_\infty + u_1(0, r_0))^2 + \frac{1}{2}u_r^2(0, r_0) \qquad (6)$$

$$\frac{1}{\rho}P(0, r_0) + \frac{1}{2}(u_\infty + u_1(0, r_0))^2 + \frac{1}{2}u_r^2(0, r_0) = \frac{1}{2}P_2(r_2) + \frac{1}{2}(u_\infty + u_1(\infty, r_2))^2 \qquad (7)$$

we can find the pressure variation for radius r_0:

$$P(+0, r_0) - P(-0, r_0) = P_2(r_2) - P_\infty + \rho\left(u_\infty + \frac{1}{2}u_1(\infty, r_2)\right)u_1(\infty, r_2) \qquad (8)$$

and the variation of specific energy for the same radius :

$$e(+0, r_0) + e(-0, r_0) = \frac{1}{\rho}(P_2(r_2) - P_\infty) + \left(u_\infty + \frac{1}{2}u_1(\infty, r_2)\right)u_1(\infty, r_2) \qquad (9)$$

It requires: $P_2(r_2) = P_\infty$. This assumption is natural for real propeller but is difficult to analyze in case of ideal propeller because it reaches the borders of the model. In that case we can write the variation of specific energy:

$$e(+0, r_0) + e(-0, r_0) = \left(u_\infty + \frac{1}{2}u_1(\infty, r_2)\right)u_1(\infty, r_2) \qquad (10)$$

Elementary turbine power is:

$$dP^T(r_0) = dm(r_0)\big(e(-0,r_0) - e(+0,r_0)\big)$$
$$= -\rho\big(u_\infty + u_1(0,r_0)\big)2\pi r_0 dr_0$$
$$* \left(u_\infty + \frac{1}{2}u_1(\infty,r_2)\right)u_1(\infty,r_2) \qquad (11)$$

Speed real movement of fluid from the propeller trail has three components: axial, radial and tangential. Momentum theory is based on the assumption that there is no tangential velocity (no movement of rotation) and replaced the propeller by an active disk. This disc produces rapid increase of pressure in fluid but not change the axial component of velocity by crossing. The described method is developed on the principles of mass, momentum and angular momentum. This method initiated by Jukovski takes into account the speed rotation of the fluid that occurs due to propeller torque (Dumitrescu et al.,1990). This rotational motion is due to additional energy losses.

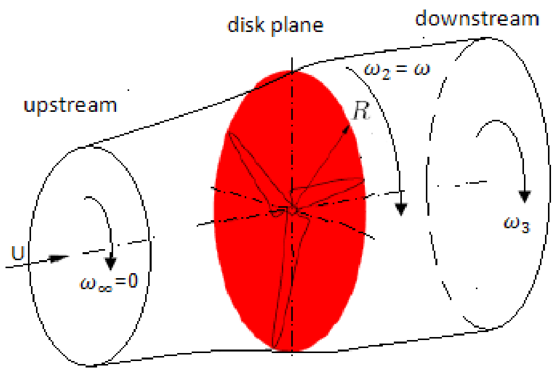

Fig.3. Angular velocity distribution

The existence of rotational motion involves filling the disk active role with the possibility to induce a tangential component of fluid velocity, other components remaining unchanged by the presence of disc.The increase in static pressure across the disc is determined by the appearance of the tangential component of velocity. If we note ω_3, angular velocity of the fluid at infinity downstream, $\omega_2 = \omega$, its value behind of the disk, and $\boldsymbol{\omega}_\infty = 0$, the angular velocity before of the disk to infinity upstream , we have a picture of the angular distribution of speeds as you can see in the figure 3. Consider an elementary current tube annular intersecting the disk as circles of radius r and r+dr, Vx and Vr are respectively the axial and radial components of fluid velocity.Noting P_1 and P_2 the static pressures on the faces of the disc with radius r, the axial velocity component V_{3x} (V_{3r}=0), and with P_3 pressure at infinity downstream to a radius r_3 (see figure 4); where r_3 is the distance between the propeller axis and the current line, who cut the disk at distance r to axis.

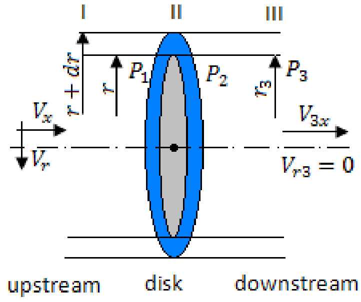

Fig.4. Basic characteristics and field

Disc disturb of uniform axial flow at infinity upstream, namely an area in front of the disk fluid velocity has a radial and axial component.
Applying the continuity equation between sections I and III

$$2\pi r_1 dr_1\, V_{1a} = 2\pi r3dr_3 V_{3a} \qquad (12)$$

if we customize our case:

$$V_x rdr = V_{3x} r_3 dr_3$$

and the angular momentum equation applied between sections II and III we have that:

$$r_1^2\omega(\mathrm{r}_1, x_1) = r_3^2\omega(\mathrm{r}_3, x_3)$$
$$\omega r^2 = \omega_3 r_3^2$$

This latter relationship represent equality of angular momentum from the active disk with surface $dS_p = 2\pi r dr$ and angular momentum at infinity downstream to take into account the continuity equation : $V_x rdr = V_{3x} r_3 dr_3$

$$dM = \rho V_x \omega r^2 dS_p \text{- for section II}$$
$$dM = \rho V_{3x}\omega_3 r_3^2 dS_3, \text{ for section III}$$
$$dS_p = 2\pi r dr \text{ and } dS_3 = 2\pi r_3 dr_3$$

Imposing equality between these two relationships have:

$$V_x \omega r^2 dS_p = V_{3x}\omega_3 r_3^2 dS_3$$

Hence considering the continuity equation:

$$\omega r^2 = \omega_3 r_3^2$$

Bernoulli`s equation applied in front and after propeller gives us:

$$h_\infty = p_\infty + \frac{1}{2}\rho V_\infty^2 = p_1 + \frac{1}{2}\rho(V_x^2 + V_r^2)$$

$$h_3 = p_2 + \frac{1}{2}\rho(V_x^2 + V_r^2 + \omega^2 r^2)$$

$$= p_3 + \frac{1}{2}\rho(V_{3x}^2 + \omega_3^2 r_3^2)$$

how we as:

$$h_3 - h_\infty = p_2 - p_1 + \tfrac{1}{2}\rho\omega^2 r^2 \qquad (13)$$

This relationship shows that the total pressure variation across the disk is larger than the static pressure jump $\Delta p = p_2 - p_1$ with a quantity that represents the kinetic energy of rotational motion appeared in the fluid due to propeller torque. From the above expressions we also:

$$p_\infty - p_3 = \frac{1}{2}\rho(V_{3x}^2 - V_\infty^2) + \frac{1}{2}\rho\omega_3^2 r_3^2 - \frac{1}{2}\rho\omega^2 r^2 - \Delta p \qquad (14)$$

Notice that, generally because of rotational movement, the pressure in the trail is less than the external pressure p_∞. Now we compare the movement to the propeller blades rotate with angular velocity Ω, then upstream up to the front disk, relative angular velocity will be Ω and decrease in Ω – ω behind the disk and Ω-ω_3 to infinity downstream.If we assume that the relative motion around the blade is the potential and conserved component velocity of the radial plane then Bernoulli`s equation gives the static pressure increase.

$$p_1 + \rho\frac{\Omega^2 r^2}{2} = p_2 + \frac{\rho}{2}(\Omega - \omega)^2 r^2$$

$$\Delta p = p_2 - p_1 = \rho\frac{\Omega^2 r^2}{2} - \frac{\rho}{2}(\Omega - \omega)^2 r^2$$

$$= \tfrac{\rho}{2} r^2\left[\Omega^2 - (\Omega - \omega)^2\right] \qquad (15)$$

$$\Delta p = \rho(\Omega - \frac{\omega}{2})\omega r^2$$

Considering this relationship and the fact that $\omega r^2 = \omega_3 r_3^2$ relationship $p_\infty - p_3$ becomes :

$$p_\infty - p_3 = \frac{1}{2}\rho(V_{3x}^2 - V_\infty^2) + \frac{1}{2}\rho\omega_3^2 r_3^2 - \frac{1}{2}\rho\omega^2 r^2 - \Delta p \qquad (16)$$

where Δp is given by relation (15).Relationship to static pressure variation of the sections (I) and (III) is:

$$p_\infty - p_3 = \tfrac{1}{2}\rho(V_{3x}^2 - V_\infty^2) - \rho\omega_3 r_3^2\left(\Omega - \tfrac{\omega_3}{2}\right) \qquad (17)$$

To calculate the torque is considered a blade element between rays r and r + dr. Below is determined forces acting on the elementary blade which are caused by hydrodynamic action of fluid flow on the propeller.

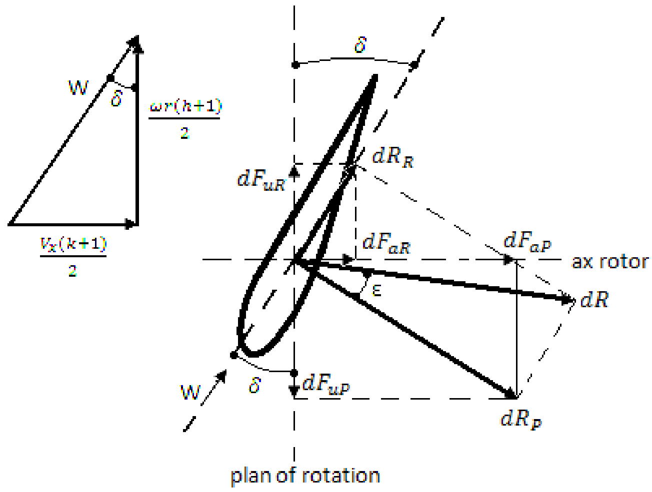

Fig.5. Blade cross-section diagram used to derive forces

Stream of fluid moving with constant velocity *W*, interacts with a blade element under a certain angle of incidence. This interaction generates a elementary hydrodynamic force *dR* which decomposes into two components elementary (drag dR_R and lift dR_P) (see figure 5).

$$dR_P = \tfrac{1}{2}\rho C_P W^2 c dr \qquad (18)$$

$$dR_R = \frac{1}{2}\rho C_R W^2 c dr$$

The decomposition of these forces in a direction parallel to the axis of the rotor giving elementary axial force dT_a

$$dT_a = dR_P cos\delta + dR_R sin\delta \qquad (19)$$

By decomposing drag and lift forces in the plane of rotation (the direction of tangential velocity) we can determine the elementary tangential force dT_u:

$$dT_u = dR_P sin\delta - dR_R cos\delta \qquad (20)$$

The blades are built on a known hydrodynamic profile and following are known profile characteristics, $C_P = C_P(\alpha)$, and, $C_R = C_R(\alpha)$. The profile coefficient μ is defined by the relationship

$$\mu = tg\varepsilon = \frac{C_R}{C_P} \qquad (21)$$

Given equation (21) relationships (19) and (20) become:

$$dT_a = \frac{\rho}{2} cdrW^2 C_P \frac{\cos(\delta-\varepsilon)}{\cos\varepsilon}$$
$$dT_u = \frac{\rho}{2} cdrW^2 C_P \frac{\sin(\delta-\varepsilon)}{\cos\varepsilon} \quad (22)$$

δ is angle between the absolute velocity W and the tangential velocity u. Consequently contribution of blade elements located between rays r and r + dr for the total axial force dT_{ta} and torque dM will be:

$$dT_{ta} = ndT_a = n\frac{\rho}{2} cdrW^2 C_P \frac{\cos(\delta-\varepsilon)}{\cos\varepsilon} \quad (23)$$

$$dM = nrdT_u = n\frac{\rho}{2} rcdrW^2 C_P \frac{\sin(\delta-\varepsilon)}{\cos\varepsilon} \quad (24)$$

In the above relations, c is chord of profile, and n is the number of blades. Power coefficient Kp is defined by the relationship:

$$K_P = \frac{dP_T}{dP_C} = \frac{cn\omega(V_\infty^2+\omega^2 r^2)}{2\pi V_\infty^3} C_P \frac{\sin(\delta-\varepsilon)}{\cos\varepsilon}$$
$$= \lambda^2(1+k)(h-1) \quad (25)$$

where dP_T is the power output of the turbine rotor located between rays r and r+dr, and dP_C is elementary power developed by fluid flow on the same surface; K is coefficient of axial velocity induced in the disk plane, and *h* is coefficient of tangential velocity induced in the disk plane. These coefficients will be determined later. It aims to optimize the power coefficient K_P. The elementary turbine shaft torque is:

$$dM = n\frac{\rho}{2} rcdrW^2 C_P \frac{\sin(\delta-\varepsilon)}{\cos\varepsilon} =$$
$$\rho\pi r^3 dr\omega V_\infty(1+k)(h-1) \quad (26)$$

where *W* is the resultant current speed, and

$$\delta = arcctg\frac{\omega r(h+1)}{V_\infty(k+1)} \quad (27)$$

E and G are dimensionless expressions :

$$E = \frac{h-1}{h+1} = \frac{C_P nc\sin(\delta-\varepsilon)}{4\pi r\cos\varepsilon\sin 2\delta} \quad (28)$$

$$G = \frac{(1-k)}{(k+1)} = \frac{C_P nc\cos(\delta-\varepsilon)}{8\pi r sin^2\delta\ \cos\varepsilon} \quad (29)$$

Making the ratio between E and G we have:

$$\frac{G}{E} = ctg(\delta-\varepsilon)ctg\delta \quad (30)$$

The parameter Mo is defined with relation:

$$M_0 = \frac{E}{\left(1+\frac{\mu}{\lambda}\right)\left(\frac{1+k}{1+h}\right)} \quad (31)$$

Finally lift coefficient C_P can be determined in a section of profile with formula:

$$C_P = \frac{8\pi r(1-k)sin^2\delta\ \cos\varepsilon}{cn(k+1)\cos(\delta-\varepsilon)} \quad (32)$$

c is the chord of the blade profile in current section. For a given value of λ, K_P power coefficient given by (25) passes through a maximum. Impose extreme condition for Kp :

$$\frac{dK_p}{dk} = 0 \quad (33)$$

Relation (33) leads to a cubic equation in respect to the variable k

$$4k^3 - 3k(\lambda^2+1) + \lambda^2 + 1 = 0 \quad (34)$$

After solving equation (34) to retain the solution that satisfies the condition $0 \le K \le 1$. With value of K thus determined is defined coefficient of the tangential velocity :

$$h = \sqrt{1+\frac{k^2}{\lambda^2}} \quad (35)$$

power coefficient is defined by the relationship :

$$K_p = \lambda^2(1+k)\left(\sqrt{1+\frac{1-k^2}{\lambda^2}} - 1\right) \quad (36)$$

For a concrete example,V=0.358 [m/s], Λ=4.6, N=0.95 [W] turbine diameter is calculated : D=0.23 [m]. Profile for achieving the blade is Eppler E 479
In the figure below you can see the variation of lift coefficient :

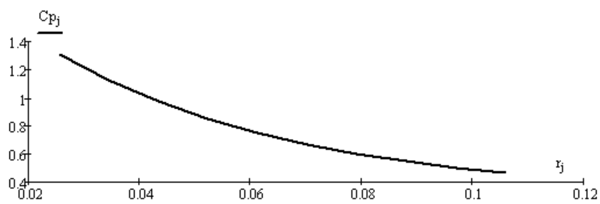

Fig.7 Lift Coefficient

Lift coefficient provides information on the performance of selected profile. Also represented in figure 8 is the variation of drag coefficient.

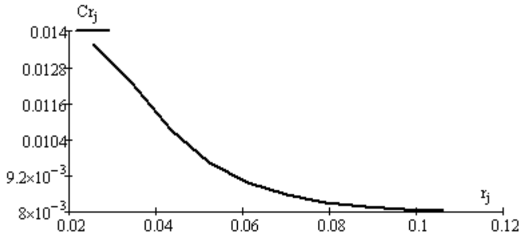

Fig. 8 Drag coefficient

As is apparent coefficient of resistance decreases with increasing radius. This leads to better aerodynamic performance. With these two coefficients is defined solidity ratio (see figure 9):

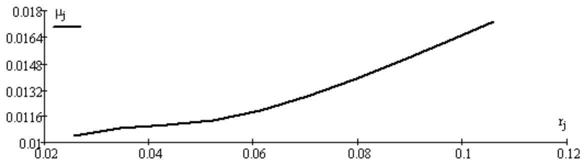

Fig. 9 Solidity profile

In the next figure we can see the variation of attack angle:

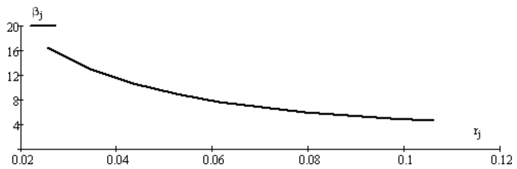

Fig. 10 Angle of Attack

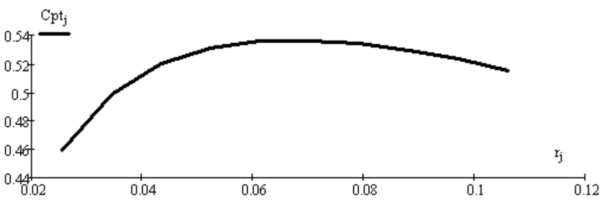

Fig. 11 Power Coefficient

Power coefficient is a measure of mechanical power transmitted from the turbine rotor for low-speed shaft.

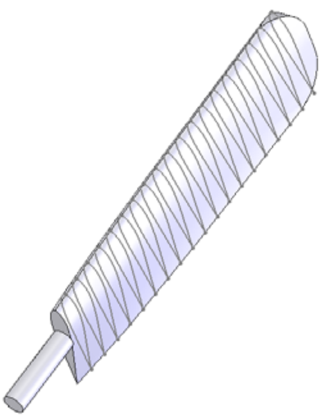

Fig. 12 Blade of the turbine

Finally obtain blade for the turbine as can be seen in figure 12.

4. CONCLUSIONS

Convert the kinetic energy of water currents, rivers or streams or the sea is a technology that does not require dams which are considered more advanced and less polluting.

Design method used for wind turbines can be successfully applied in the case of water current turbines.

Turbine design has been achieved considering constant fluid speed. Refer to the analysis and design studies on the possibility of hydrodynamic parameters for variable speed water.

REFERENCES

1. Dumitrescu, H.; Georgescu A.; s.a.(1990). *Calculul elicei,* Romanian Academy Publishing House, ISBN 973-27-0053-X, Bucharest.
2. Narayana, M., Sugathapala, A.,(2004). *Performance of battery-charging wind turbine developed by the nerd centre*, Proceedings of the Tenth Asian Congress of Fluid Mechanics, vol.11, pp.17-21, Peradeniya Sri Lanka.
3. Zhiquan, Y., Yan, C., Feng, Z., (2000). *HAWT Aerodynamic Performance Prediction using a Semi-Rigid Wake Model Based on Vortex Theory*, Journal of Wind Engineering, 24, pp 349-359.
4. Lanzafame, R., Messina, M. (2007). *Fluid dynamics wind turbine design: Critical analysis, optimization and application of BEM theory*, Available from: http://www.sciencedirect.com, Accessed: 10/09/2010.
5. Kishinami, K., Taniguchi, H. et al. (2005), *Theoretical and experimental study on the aerodynamic characteristics of a horizontal axis wind turbine,* Journal of Energy, 30, pp. 2089–2100.
6. Calarasu, D, Scurtu, D., Ciobanu, B., (2010). *Analysis of specific phenomena occurring during the operation of a closed hydraulic cylinder,* Proceedings of ModTech2010-New Face of TMCR International Conference, ISSN 2066-3919, pp. 163-167
7. Belgiu, G., et al. (2010). *Manufacturing process management using a CAD/CAM/CAE success operation prediction*, Proceedings of ModTech2010-New Face of TMCR International Conference, ISSN 2066-3919, pp. 115-119
8. Berbinschi, S., Teodor, V., Oancea, N., (2010). *Kinematical method for rack-gear tool's profiling in CATIA design environment*, Int. J. Modern Manufacturing Technologies II(2), ISSN 2067-3604, pp. 23-31, Iasi.

Received: March 05, 2011 / Accepted: May 30, 2011 / Paper published online: June 10, 2011