ROBUST COORDINATED PRODUCTION AND TRANSPORTATION SCHEDULING ON TWO MACHINE FLOW SHOP USING CONSTRAINT EQUATIONS

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Abstract: Coordinated scheduling of production and transportation is significant to slim down overall cost of production, supplement the level of service and efficiency. However uncertainties in production and transportation time originate deviations in concrete performance of jobs schedule. Thus, the risk for achieving inferior performance of schedule is an indispensable assessor for determining the efficiency of schedule. To contend with the risk, a constraint based robust coordinated schedule is projected to condense the risk of exceeding total flow-time from a firm limit in two-machine flow-shop, where total flow-time is reckon as a performance evaluator. The presented course of action for coordinated scheduling is applied in two-machine flow-shop of a manufacturing company in China. Results divulge that the anticipated schedule presents maximum probability to attain total flow-time less than a threshold and the schedule has minimum risk of escalating total flow-time from a certain limit.
Keywords: Coordinated scheduling, Risk, robust schedule, Schedule performance, Total flow time

1. INTRODUCTION

Coordinated scheduling of production and transportation is meaningful to shrink overall cost, augment efficiency and customer service level (Renato de Matta and Tan Miller, 2004), (Tadeusz Sawik, 2009), (Chen, Z.-L. and Vairaktarakis, G.L., 2005) (Wang, H and Lee, C.-Y., 2005).
Transporting facility is desired to shift jobs between the individual processes of supply chain, thus, transportation time deliberation is compulsory in synchronized scheduling of the processes. This certainty has forced researchers to mull over transportation in the sequencing of jobs in flow shops (Lee, C. Y., and Chen, Z. L., 2001), (Lixin Tang et al., 2009), (Li, K.P et al., 2005), (Li, K.P et al., 2006). However, they have considered deterministic environment in their analysis. Nevertheless, in real time different sources of uncertainties can arise; durations may not be known, resources may have lower capacity than expected (i.e., machine breakdown), new tasks may be taken into account etc. In these circumstances, deterministic schedules are not feasible to handle uncertain events and dose not give accurate results.
Cyril Briand et al. (2007) presented reactive and proactive approaches to handle uncertain situation. S. Van de Vonder et al. (in press) indicated that a proactive or robust approach is more effective than a pure reactive approach. Robust schedule do better, presents quality of results and are able to tolerate small deviations (Mikkel T. Jensen, 2003).
Thus robust approach which considers stability of schedule in uncertain environment has been extensively studied (Wu, S.D etal., 1993), (R. O'Donovan et al., 1999), (Akturk MS and Gorgulu E., 1999), (Bean JC et al., 1991), (Rangsaritratsamee R., et al., 2004). However, robust schedules considering improvement in quality and performance level of schedule in the presence of uncertainties in flow shops has been studied a little. R. L. Daniels and P. Kouvelis (1997) presented a β-robust schedule for single machine environment to minimize the risk of exceeding total flow time from a firm limit considering uncertainty in processing time. Christine et al. (2009) focused on the same objective for single machine and modeled robust schedule with constraints equations. Kouvelis et al. (2000) studied robust scheduling in considering the risk of deprived system performance due to uncertainty in processing times for two-machine flow-shop. However, to form efficient jobs schedule, it is not adequate to consider the risk for measuring schedule robustness due to uncertainty in processing times. However, there is much consequence of reducing the risk of substandard performance or increasing the probability to achieve assured performance level. Thus, Saif Ullah et al. (2009) quantified the risk in order to get the most approximate jobs completion time. They considered total flow-time as a performance measure of the schedule and presented robust schedule on two machine flow shop to achieve definite performance level of schedule not worse than a firm limit.
However, most of the literature regarding robust scheduling has not considered transportation facility in flow shops. In order to formulate efficient jobs schedule, minimize overall cost and jobs lateness, it

is pleasing to consider robust scheduling considering coordination of production and transportation facilities. This persuade us to commence transportation facility in two machine flow shop and present β-robust schedule with transportation time consideration, to gain actual performance level of schedule not worse than a certain limit and to minimize the risk of exceeding total flow time from a threshold.

Rest of the paper is organized as follows. Section 2 explains the literatures' work corresponding to our research. Section 3 identifies total flow-time for two machine flow-shop. Section 4 describes the constraints, final sequence of jobs and indicates the probability of getting total flow-time not less than firm limit. Section 5 presents the data examination and results of a manufacturing company in China. Section 6 shows conclusion and future path of the research.

2. RELATED WORK

For β-robust scheduling Christine et al. [18] presented the probability $(flowtime(s) \leq S)$ for a jobs sequence as indicated in equation (1).

$$probability(flowtime(s) \leq S) = \frac{1}{2} + \varphi(Z) \quad (1)$$

Where, $S \geq 0$ and Z is a normally distributed random variable as described in equation (2).

$$Z = \frac{S - mean(flowtime)}{\sqrt{variance(flowtime)}} \quad (2)$$

$\varphi(Z)$ is obtained from an estimate given by [16] shown in equation (3).

$$\varphi(Z) \approx \phi(Z) = \begin{cases} 0.1Z(4.4 - Z) & (0 \leq Z \leq 2.2) \\ 0.49 & (2.2 < Z < 2.6) \\ 0.50 & (Z \geq 2.6) \end{cases} \quad (3)$$

The objective is presented in equation (4).

$$objective = \max(probability(flowtime(s) \leq S)) = \frac{1}{2} + \varphi(Z) \quad (4)$$

3. TOTAL FLOW TIME

Total flow-time is the summation of end time of all jobs. Fig. 1 describes the end time of jobs which is the time when job completes its processing on all machines in the flow-shop. End time is dependent of waiting, transportation and processing times of the jobs. However, in flow shop the waiting time of jobs on transporter and machines is sequence dependent. Thus, more than one variable are considered and more than one conditional equation are used to calculate end time of jobs in two machine flow-shop, which are described in this section both for certain and uncertain processing and transportation times.

3.1 Total flow-time for certain processing times

n jobs are considered to process on both machines M_1 and M_2. Two transporters are used to move semi finished jobs from M_1 to M_2. Flowchart shown in Fig. 2. describes the end time of jobs in flow-shop. Arrival time of all jobs, start time of first job on M_1 and the waiting time of first job on transporter and machines are assumed to be zero. There is no idle time on M_1 and all jobs complete processing on M_1 before arriving on M_2. Setup times are also included in the processing times of jobs. Loaded transporter moves from M_1 to M_2, at the same time the unloaded transporter travels from M_2 to M_1. Both transporters take equal time for this movement for a particular job. Loading and unloading times of jobs are included in the transportation times, thus transportation time for each job is different.

Variables

r is a positive real number such that $r \leq n$. k describes all the given n jobs $\{1,2,3,....,n\}$ and j indicates current job. T designates transporter and T_j is the transportation time of transporter from M_1 to M_2 or from M_2 to M_1. t_{j1}, t_{j2} are the processing times of job j on M_1 and M_2 respectively. C_j is the completion time of job j on M_1, D_j is the delivery time of semi finished job j from M_1 to M_2. E_j is the end time of job j. TFT represents the total flow-time of jobs.

$\forall \; 0 < j \leq n$; equation (5) gives the completion time of jobs on M_1, equation (6) and (7) represents delivery time of semi finished jobs to M_2 and end time of first job respectively.

$$C_j = \sum_{r=1}^{j} t_{r1} \quad (5)$$

$$D_1 = C_1 + T_1 \quad (6)$$

$$E_1 = D_1 + t_{12} \quad (7)$$

From $\forall \; 1 < j \leq n$ if, $C_j \geq D_{(j-1)}$ then, job j do not have to wait for transporter T. Delivery time to M_2 of all such jobs is described in Equation (8).

$$D_j = \sum_{r=1}^{j} t_{r1} + T_j \quad (8)$$

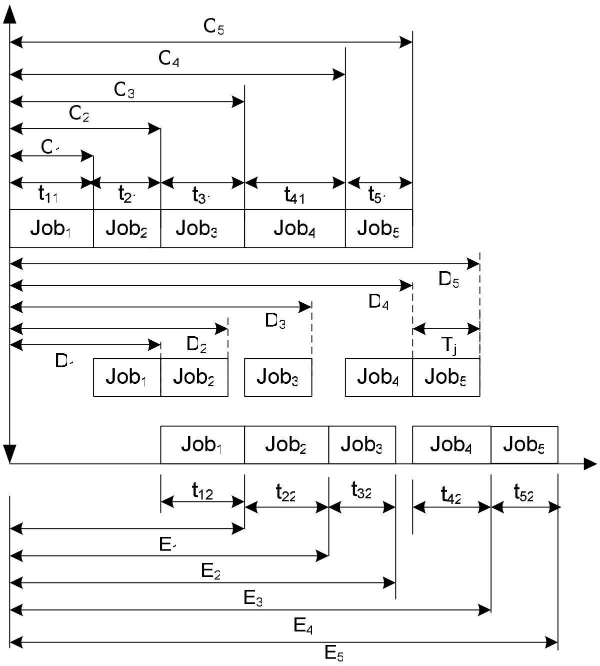

Fig.1. Completion time, delivery time to M_2 and end time of jobs

From $\forall \ 1 < j \le n$ if, $D_j \ge E_{(j-1)}$ then, job j have not to wait on M_2 after its delivery to M_2. Equation (9) shows end time of all jobs which do not have to wait for T.

$$E_j = D_j + t_{j2} \tag{9}$$

From $\forall \ 1 < j \le n$ if, $C_j < D_{(j-1)}$ then, job j have to wait for T. Equation (10) shows delivery time for jobs on M_2 which have to wait for T. In equation (10) x is a positive integer obtained from the flowchart shown in Fig. 2. x is used to represent a job from $\forall \ 0 < x < j$ which have not waited for T and is located earlier to job j in the schedule.

$$D_j = \sum_{r=1}^{x} t_{r1} + \sum_{r=x}^{j} T_j \tag{10}$$

From $\forall \ 1 < j \le n$ if, $D_j < E_{(j-1)}$ then, job j have to wait on M_2 after its delivery to M_2. Equation (11) shows end time for jobs which have to wait on M_2. In equation (11) y is a positive integer obtained from the flowchart shown in Fig. 2. y is used to represent a job from $\forall \ 0 < y < j$ which have not waited on M_2 and is located earlier to job j in the schedule.

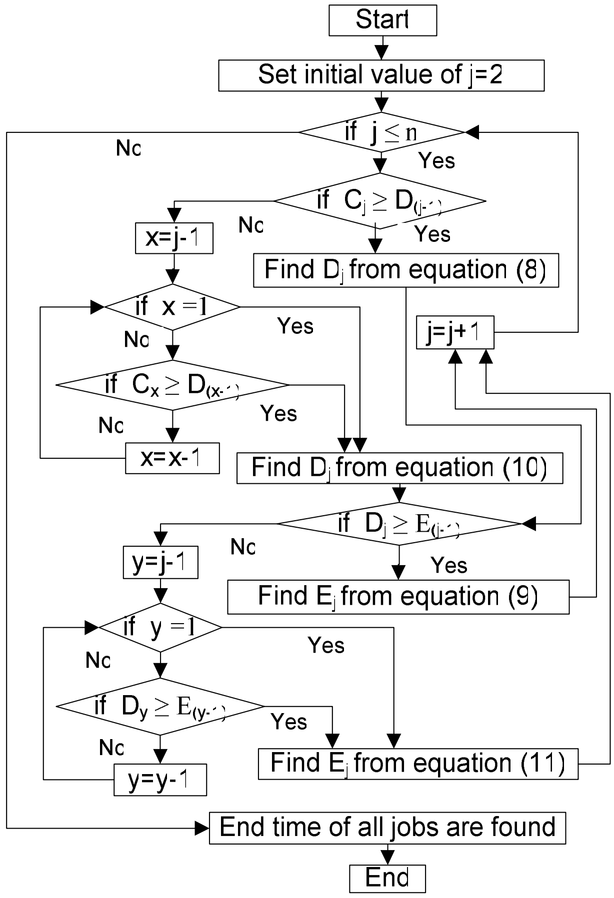

Fig.2. Flow chart showing end time of jobs

Total flow-time is illustrated in Equation (12).

$$E_j = \sum_{r=1}^{y} t_{r2} + D_y \tag{11}$$

$$TFT = \sum_{j=1}^{n} E_j \tag{12}$$

3.2 Total flow-time for uncertain processing

In real examples, the transportation and processing times of jobs varies. Therefore, it is considerable to reflect on processing and transportation time of each job as a random variable. Suppose initial jobs sequence is given and $\forall \ 0 < j \le n$; processing and transportation times of jobs on both machines and transporter are assumed to be independent normally distributed variables. To make final jobs schedule and to find total flow-time of the given and the final schedule, following variables are considered.

Variables

$\mu_{j1}, \mu_{j2}, \mu_{jT}$ and $\mu_{k1}, \mu_{k2}, \mu_{kT}$ are the mean value of processing times of job j and job k on M_1, M_2 and transporter respectively. $\sigma_{j1}^2, \sigma_{j2}^2, \sigma_{jT}^2$ and $\sigma_{k1}^2, \sigma_{k2}^2, \sigma_{kT}^2$ are the variance processing time of job j and job k on M_1, M_2 and transporter respectively. pos_j and pos_k are positive integers describing the positions of job *j* and job *k* in the schedule. *i* is a positive variable i.e. $0 < i \le n$, and is used to define position of job *j*, in the final schedule. TFT_g, MFT_j and VFT_j are the total flow-time, mean flow-time and variance flow-time for given jobs schedule respectively. TFT_i, MFT_f and VFT_f are the total flow-time, mean flow-time and variance flow-time of final jobs schedule respectively. Set g is used to indicate set of jobs which are positioned in final schedule, initially set g is empty.

From the proposed assumption, processing times on machines and transporter are normally distributed and described presented as $t_{j1} \sim N(\mu_{j1}, \sigma_{j1}^2)$, $t_{j2} \sim N(\mu_{j2}, \sigma_{j2}^2)$ and $T_j \sim N(\mu_{jT}, \sigma_{jT}^2)$ respectively. However, sum of two independent normally distributed variables is also normally distributed variable (2007). Since the processing and transportation times are independent normally distributed random variables, and completion time and end time of jobs are obtained by adding transportation and processing times. Thus, $C_j \sim N(\mu_{cj}, \sigma_{cj}^2)$, $D_j \sim N(\mu_{dj}, \sigma_{dj}^2)$ and $E_j \sim N(\mu_{Ej}, \sigma_{Ej}^2)$ gives completion time, delivery time of semi finished jobs to M_2 and end time of job *j* respectively. Where, $\mu_{cj}, \mu_{dj}, \mu_{Ej}, \sigma_{cj}^2, \sigma_{dj}^2$ and σ_{Ej}^2 represents mean value of completion time, mean value of delivery tome to M_2, mean value of end time, variance of completion time, variance of delivery time to M_2 and variance of end time for job j respectively. Equations (13), (14) and (15) are obtained from equation (5), (6) and (7).

$$C_j \sim N\left(\sum_{r=1}^{j}\mu_{r1}, \sum_{r=1}^{j}\sigma_{r1}^2\right) \qquad (13)$$

$$D_1 \sim N(\mu_{11} + \mu_{1T}, \sigma_{11}^2 + \sigma_{1T}^2) \qquad (14)$$

$$E_1 \sim N(\mu_{11} + \mu_{1T} + \mu_{12}, \sigma_{11}^2 + \sigma_{1T}^2 + \sigma_{12}^2) \quad (15)$$

Delivery time to M_2 and end time corresponding to equation (8) and (9) are described in equation (16) and (17) respectively.

$$D_j \sim N\left(\left(\sum_{r=1}^{j}\mu_{r1} + \mu_{jT}\right), \left(\sum_{r=1}^{j}\sigma_{r1}^2 + \sigma_{jT}^2\right)\right) \quad (16)$$

$$E_j \sim N\left((\mu_{dj} + \mu_{j2}), (\sigma_{dj}^2 + \sigma_{j2}^2)\right) \quad (17)$$

Delivery time to M_2 and end time corresponding to equation (10) and (11) are indicated in equation (18) and (19) respectively.

$$D_j \sim N\left(\left(\sum_{r=1}^{x}\mu_{r1} + \sum_{r=x}^{j}\mu_{jT}\right), \left(\sum_{r=1}^{x}\sigma_{r1}^2 + \sum_{r=x}^{j}\sigma_{jT}^2\right)\right) \quad (18)$$

$$E_j \sim N\left(\left(\sum_{r=1}^{y}\mu_{r2} + \mu_{dy}\right), \left(\sum_{r=1}^{y}\sigma_{r2}^2 + \sigma_{dy}^2\right)\right) \quad (19)$$

From equation (12) total flow-time is the sum of end time of all jobs. Since end times of jobs are normally distributed variables therefore, total flow-time is also a normally distributed variable. Equation (20) and (21) shows normally distributed total flow time for given and final schedule respectively.

$$TFT_g \sim N\left(\sum_{j=1}^{n}\mu_{Ej}, \sum_{j=1}^{n}\sigma_{Ej}^2\right) \qquad (20)$$

$$TFT_f \sim N\left(\sum_{i=1}^{n}\mu_{Ei}, \sum_{i=1}^{n}\sigma_{Ei}^2\right) \qquad (21)$$

4. FINAL SCHEDULE AND PROBABILITY

To formulate β-robust schedule the constraint equations are given in this section.

4.1 Positional constraints

The given sequence of jobs may not have maximum probability of gaining total flow-time less than a certain limit. For optimal schedule the positional relation between jobs is to be known. Three constraints are described. First and second constraints are obtained by extending the constraints given by Christine et al. (2009). Relative position of each job j with respect to job k is obtained when any one of the constraint is satisfied. Third constraint is determined by simultaneously solving equation (1) and (3). From $\forall \quad 0 < j < k \le n$, constraint equations determine the positioning of job j relative to each job k. Thus $(n - j)$ number of relations are obtained for every job j, each relation showing position of job j relative to each job k. $\forall \; 0 < j < k \le n$

First constraint

If, $((\mu_{j1} \le \mu_{k1}) \wedge (\sigma_{j1}^2 \le \sigma_{k1}^2)) \;\wedge$
$((\mu_{jT} \le \mu_{kT}) \wedge (\sigma_{jT}^2 \le \sigma_{kT}^2)) \;\wedge$
$((\mu_{j2} \le \mu_{k2}) \wedge (\sigma_{j2}^2 \le \sigma_{k2}^2))$.

Then, $pos_j < pos_k$. Otherwise, second constraint is applied.

Second constraint

If, $((\mu_{j1} \geq \mu_{k1}) \wedge (\sigma_{j1}^2 \geq \sigma_{k1}^2)) \wedge$
$((\mu_{jT} \geq \mu_{kT}) \wedge (\sigma_{jT}^2 \geq \sigma_{kT}^2)) \wedge$
$((\mu_{j2} \geq \mu_{k2}) \wedge (\sigma_{j2}^2 \geq \sigma_{k2}^2))$.

Then, $pos_j > pos_k$. Otherwise, third constraint is used.

Third constraint

For a jobs schedule $\varphi(Z)$ value and probability of $(flowtime \leq S)$ are directly related which is observed by analyzing equation (1) and (3).

Given position of job j is temporarily interchanged with job k, when first and second constraints are failed. Comparing $\varphi(Z)$ value of given schedule $\varphi(Z_g)$, and $\varphi(Z)$ value of temporarily changed schedule $\varphi(Z_{ch})$, determines the comparison of probability of $(flowtime(s) \leq S)$ for the given schedule and temporarily changed jobs schedule. Therefore, to maximize the probability of flow-time being less than a threshold, subsequent constraint is also considered.

$$\text{If, } \varphi(Z_g) \geq \varphi(Z_{ch}). \text{ Then, } pos_j < pos_k$$
$$\text{else, } pos_j > pos_k$$

After getting the relative positioning of job j with job k, the temporary change in schedule is reversed to maintain jobs positioning in the original jobs schedule.

4.2 Sequencing of jobs for final schedule

$\forall\ 0 < j < k \leq n$, constraint equation gives $(n-j)$ number of positional relations of job j with all k jobs. These relative positions of all jobs are helpful for assigning their positions in the final schedule. $\forall\ 0 < k \leq n$ when job j from the given set of jobs gives $pos_j < pos_k$ for all k jobs, the position of job j is stored first in the final schedule. Similar comparison determines the job to be assign on second position in the final schedule and so on. Quick sort algorithm given by C.A.R. Hoare (1962) is helpful to sort jobs to form final schedule by comparing relative positions of all jobs, then final schedule is stored in set g.

4.3 Finding probability

Probability of total flow-time being less than provided limit for the final schedule is obtained from equations (1), (2), (3) and (21).

5. EXPERIMENTAL DATA ANALYSIS

Table 1 shows mean and variance of transportation and processing times of different metal pipes processed on pipe cutting and bending machines in Midea Centeral Air conditioner Manufacturing Company in China.

Uncertainties in the setup times of jobs cause variation in total flow time. Total flow-time limit for jobs is 160. Projected β-robust schedule is obtained from the given sequence using proposed equations in Turbo C.

Table 2 displays make-span, total flow-time, variance flow-time and probability of getting total flow-time less than given limit both for the given and β-robust schedule.

Table 1. Normally distributed processing times of the pipes

Job	Processing times on machines and transporter		
j	M_1 $(\mu_{j1}, \sigma_{j1}^2)$	T $(\mu_{jT}, \sigma_{jT}^2)$	M_2 $(\mu_{j2}, \sigma_{j2}^2)$
1	(8, 1.5)	(1.5, 1)	(10, 2)
2	(7, 2)	(2, 1.5)	(11, 2.5)
3	(7, 2)	(1.5, 1)	(9, 1)
4	(6, 1)	(1.5, 1)	(8, 1.5)
5	(5, 1)	(1, 1)	(7, 2)

Table 2. Comparison of the results between given schedule and the β-robust schedule

	Given Schedule	β-robust Schedule
Jobs sequence	1,2,3,4,5	5,4,3,1,2
Make-span	54.5	51
Total (flow-time)	191.5	155
Variance(flow-time)	43	35.5
Probability	0 %	80 %

Final results corresponding to the given schedule shows lot of advantages of the presented model. Result reveals that proposed schedule gives minimum make-span, total flow-time and variance. The presented β-robust schedule also maximizes the probability (i.e. 80 %) of achieving total flow-time less than the provided limit (i.e. 160), hence minimizes the risk of exceeding total flow-time from the given limit.

6. CONCLUSIONS

Coordinated scheduling problem of two-machine flow shop with consideration of uncertainty in transportation and processing times is studied to obtain a coordinated robust schedule that can minimize the risk of exceeding total flow-time of a schedule from a threshold. Constraint based schedule is anticipated for an objective of optimal minimization of risk.

Turbo C is used for solving the constraint equations to get β-robust schedule which is experimented on Midea Centeral Air conditioner Manufacturing Company in China. End result illustrates make span, total flow-time, variance flow-time and probability of getting total flow-time less than a limit, for the given and β-robust schedules. Result point out that the acquired schedule confers maximum probability to achieve total flow-time less than a threshold, hence curtails the risk of exceeding flow-time from a specified limit.

Presented research can be extended for similar objective for more than two machines of same or different jobs sequence. Research can also be extended for coordinated supply chain robust scheduling by considering number of tardy jobs, earliness, lateness, on time delivery or certain service level as to quantify the performance. First round result points out that the anticipated work can help out for such cases.

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