

THE ANALYSIS OF VIBRATING SYSTEMS BASED ON THE EXACT AND APPROXIMATE METHOD

Andrzej Buchacz¹ & Marek Płaczek²

^{1,2} The Silesian University of Technology, Faculty of Mechanical Engineering,
Institute of Engineering Processes Automation and Integrated Manufacturing Systems,
18A Konarskiego Street, Gliwice 44-100, Poland

Corresponding author: Marek Płaczek, marek.placzek@polsl.pl

Abstract: The main aim of this work is to present results of the mechanical and mechatronic system's analysis based on the exact Fourier's and approximate Galerkin's methods. The considered mechanical system is the flexural vibrating one-dimension bending beam. The considered mechatronic system is the same bending beam with the piezoelectric transducer bonded to the beam's surface. The external shunting circuit was adjoined to the transducer's clamps in order to damp system's vibration. The exact and approximate method were used to assign the dynamic flexibility of the considered mechanical system and results of this work were juxtaposed to verify the approximate method's accuracy. The correction coefficients were introduced into the approximate method to unify results of both methods. Verification of the approximate method was the first step to the further author's work connected with the mechatronic system's analysis. The aim of this work was to check accuracy of the approximate method and verify if this method may be used to mechatronic system's analysis, where it is impossible to use the exact method.

Key words: Vibration, damping, sensor, actuator, smart materials.

1. INTRODUCTION

This paper refers to the use of the approximate Galerkin's method in the one-dimension vibrating mechatronic system investigation, but especially to the verification of this method. The considered mechatronic system is the cantilever bending beam with the piezoelectric transducer bonded to the beam surface by the glue layer. Such kind of systems, including piezoelectric materials used as sensors or actuators for stabilization and damping of mechanical vibration are getting more and more popular. Piezoelectric transducers with external electric circuit can be applied in many mechanical systems such as a beam or a shaft in order to obtain required dynamic characteristic of designed system. Therefore engineers have to use very precise mathematical model of such mechatronic systems to obtain required dynamic parameters of this systems. It is indispensable to take into account geometrical and material parameters of all system's components because the omission of the influence of one of them

results in inaccuracy in the analysis of the system. In the previous authors' works, it was proved that the glue layer's geometrical and material parameters have strong influence on the dynamic flexibility of the considered mechatronic system [1-6]. The same results were obtained by Tylikowski and Pietrzakowski [10,11,12].

General, the continuous or discrete – continuous mathematical models were used by authors who were investigated such kind of systems. In the previous authors' work mechanical system was modelled by continuous mathematical model and piezoelectric transducer by discrete model. The main aim was to assign the dynamic flexibility of the system and specify the influence of the geometrical and material parameters, of all system's components on it.

It is obvious, that not only very precise mathematical model of the system is important, but very precise method of the system's analysis is important, too. It is impossible to use the exact method to analyse the considered mechatronic system. This is why the approximate method was used. The first authors' step was verification of the approximate method to check accuracy of this method and verify if it can be used to mechatronic system's analysis, where it is impossible to use the exact method.

2. CONSIDERED SYSTEMS

The considered mechatronic system is the cantilever beam which has a rectangular constant cross-section, length l and Young's modulus E . The piezoelectric transducer of length l_p and thickness h_p is bonded to the beam's surface by the glue layer of finished thickness h_k and Kirchhoff's modulus G . The glue layer has homogeneous properties in overall length. The external shunting circuit is adjoined to the transducer's clamps. This system is loaded with harmonic force $F(t)$.

The beam's vibrations affect the piezoelectric transducer which generates electric charge and produces additional stiffness of electromechanical nature, dependent on the capacitance of the transducer and adjoined external circuit. The external

circuit introduces electric dissipation resulting in electronic damping of vibration [1-6, 7, 8, 9]. Fig. 1 presents this system.

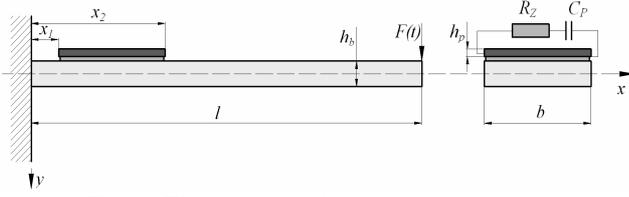


Fig. 1. The considered mechatronic system

To verify the approximate method, first, the mechanical system was being taken into consideration. The mechanical system is exactly the same beam without piezoelectric transducer and external circuit. This system is also loaded with harmonic force $F(t)$. It is shown in Fig. 2.

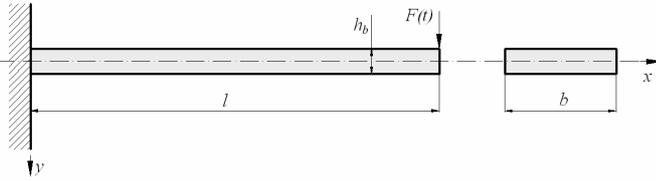


Fig. 2. The mechanical system

3. THE DISCRETE – CONTINUOUS MATHEMATICAL MODEL OF CONSIDERED SYSTEMS

In both considered systems the equation of the flexural beam vibration forced by the external force was assigned in accordance with Bernoulli – Euler's model of the beam. Displacement of the beam's cross sections was described by coordinate $y(x,t)$. Dynamic equation of motion of the beam was assigned on the basis of elementary beam and transducer's section dynamic equilibrium in agreement with d'Alembert's principle, on the assumption about the uniaxial, homogeneous strain of the transducer [1-5,10-12].

In the mechatronic system's mathematical model the equation of the piezoelectric transducer with external circuit was expressed as an equation of the linear RC circuit with harmonious voltage source [1-6, 8]. The piezoelectric transducer with external shunting circuit was treated as linear RC circuit as it is shown in figure 3.

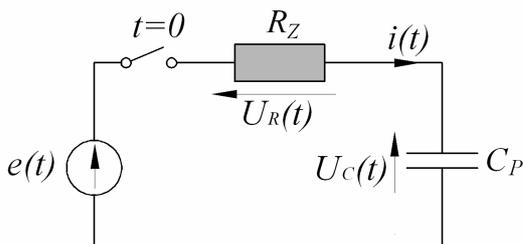


Fig. 3. Scheme of the piezoelectric transducer with external shunting circuit

R_Z is the external electric circuit resistance and C_P is the capacitance of the piezoelectric transducer.

There was the assumption about the uniaxial, homogeneous strain of the transducer and pure shear of the connection layer. The Heaviside's function $H(x)$ was introduced to curb the working space of the transducer to partition from x_1 to x_2 [1-6, 8].

4. THE MECHANICAL SYSTEM

First, the well known exact Fourier method was used to assign the dynamic flexibility, assigned Y , of the considered system, in accordance with its definition:

$$y(x,t) = Y \cdot F_0 \cos(\omega t). \quad (1)$$

The system's dynamic flexibility was assigned on the free end of the beam ($x=l$).

In the exact Fourier's method the beam's differential motion's equation is defined as a product of time and displacement's eigenfunctions:

$$y(x,t) = X_n(x) \cdot T(t), \quad (2)$$

where:

$$X_n(x) = [-\cosh(\lambda_n l) - \cos(\lambda_n l)] \cdot [\sinh(\lambda_n x) - \sin(\lambda_n x)] + [\sinh(\lambda_n l) + \sin(\lambda_n l)] \cdot [\cosh(\lambda_n x) - \cos(\lambda_n x)], \quad (3)$$

$$T(t) = A \cos(\omega_n t), \quad (4)$$

$$\lambda_n = \frac{2n-1}{2l} \pi. \quad (5)$$

Boundary conditions:

$$y(0,t) = 0, \quad (6)$$

$$\frac{\partial y(0,t)}{\partial x} = 0, \quad (7)$$

$$\frac{\partial^2 y(l,t)}{\partial x^2} = 0, \quad (8)$$

$$\frac{\partial^3 y(l,t)}{\partial x^3} = 0. \quad (9)$$

Initial conditions:

$$y(x,0) = \varphi(x), \quad (10)$$

$$\frac{\partial y(x,0)}{\partial x} = \psi(x). \quad (11)$$

Next, the dynamic flexibility of the considered mechanical system was assigned on the basis of the approximate Galerkin's method. The solution of the

beam's differential motion's equation was defined as a product of time and displacement's eigenfunctions:

$$y(x, t) = A \sum_{n=1}^{\infty} \sin \left[(2n-1) \cdot \frac{\pi}{2l} \cdot x \right] \cdot \cos(\omega t) \quad (12)$$

which meet defined boundary conditions:

$$y(0, t) = 0, \quad (13)$$

$$y(l, 0) = A. \quad (14)$$

In the approximate method the beam's deflection (that is exactly the solution of the beam's differential motion's equation) was assume as a quite simply equation, that fulfil only two boundary conditions, while in the exact method equation of the beam's deflection is very complicated and it fulfils four boundary conditions and two initial conditions. Obtained dynamic characteristic assigned on the basis of both methods are presented in the Fig. 4.

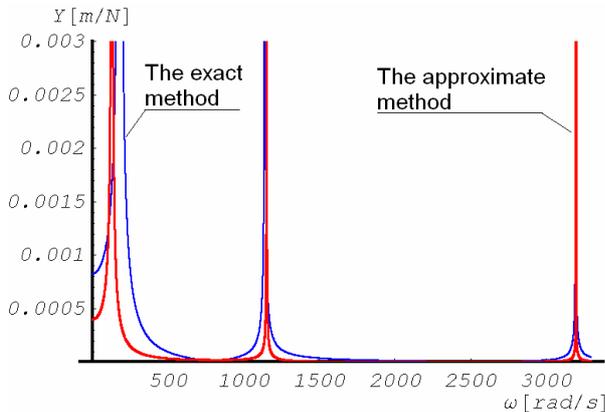


Fig. 4. The dynamic flexibility of the mechanical system

5. METHOD'S VERIFICATION

To verify the accuracy of the approximate method results of the exact and approximate method were juxtaposed on the charts for particular values of the system's periodicity.

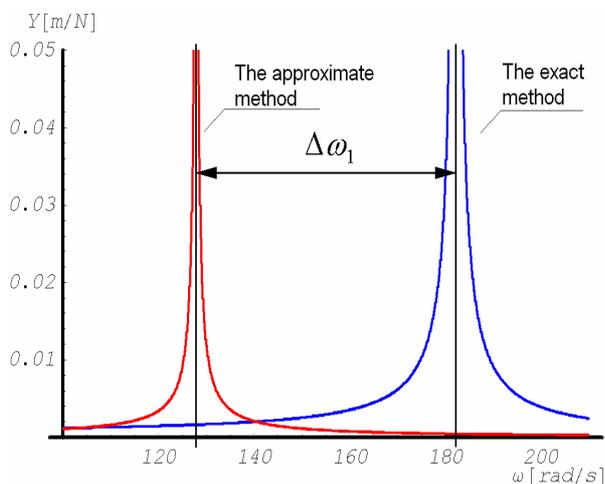


Fig. 5. The dynamic flexibility of the mechanical system, for $n=1$

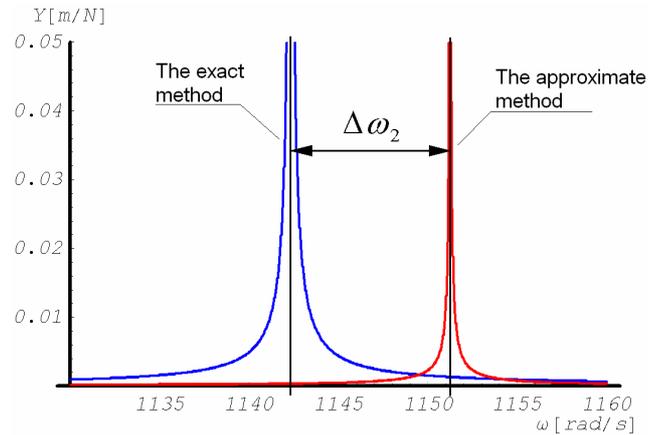


Fig. 6. The dynamic flexibility of the mechanical system, for $n=2$

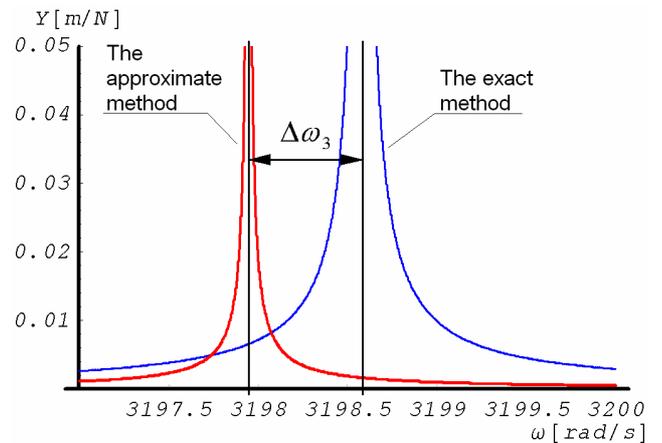


Fig. 7. The dynamic flexibility of the mechanical system, for $n=3$

Inexactness of the approximate method is very meaningful for the first three modes. They are presented in table 1.

Table 1. System's periodicity values and correction coefficients

n	The exact method [rad/s]	The approximate method [rad/s]	$\Delta\omega_n$ [rad/s]
1	$\omega_1 = 182,281$	$\omega_1 = 127,918$	$\Delta\omega_1 = 54,363$
2	$\omega_2 = 1142,34$	$\omega_2 = 1151,26$	$\Delta\omega_2 = -8,92$
3	$\omega_3 = 3198,59$	$\omega_3 = 3197,95$	$\Delta\omega_3 = 0,64$
4	$\omega_4 = 6267,98$	$\omega_4 = 6267,98$	$\Delta\omega_4 = 0$

Inexactness of the approximate method was reduced by the correction coefficients introduced for the first three values of periodicity of the system. Values of those coefficients were assigned in accordance with equation:

$$\Delta\omega_n = \omega_n - \omega_n' \quad (15)$$

where ω_n' is the value of periodicity assigned on the basis of the approximate method. There are not coefficients for the higher modes, because obtained results are the same for both methods.

The reason for this is that in the exact method values of periodicity for $n < 3$ are assigned on the basis of equations:

$$\left(\frac{1,8751}{l}\right)^2 \sqrt{\frac{E_b J_b}{\rho_b A_b}}, \text{ for } n=1, \quad (16)$$

$$\left(\frac{4,6941}{l}\right)^2 \sqrt{\frac{E_b J_b}{\rho_b A_b}}, \text{ for } n=2, \quad (17)$$

$$\left(\frac{7,85477}{l}\right)^2 \sqrt{\frac{E_b J_b}{\rho_b A_b}}, \text{ for } n=3. \quad (18)$$

Values of the system's periodicity for $n > 3$ are assigned on the basis of the same equation for both methods:

$$\omega = \sqrt{\frac{E_b J_b}{\rho_b A_b}} \cdot \lambda^2. \quad (19)$$

λ is described by the equation (5).

What is more, values of the dynamic flexibility for both methods are very similar, so implementation of the correction coefficients makes the approximate method precise enough to use it for the mechatronic systems analysis and obtain very precise results.

The dynamic flexibility assigned on the basis of the corrected approximate method for the first three modes is presented in figures 8 to 11. It is juxtaposed with the dynamic flexibility assigned on the basis of the exact method.

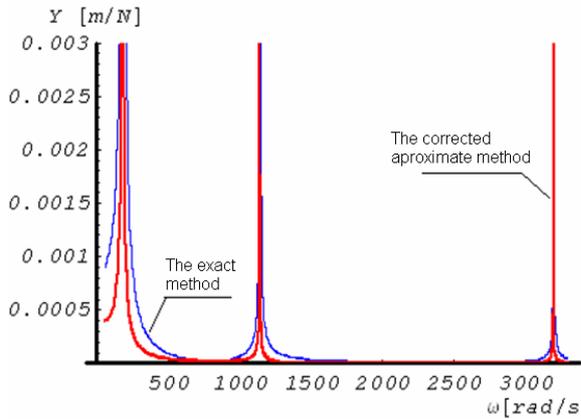


Fig. 8. The dynamic flexibility of the mechanical system, for $n=1,2,3$

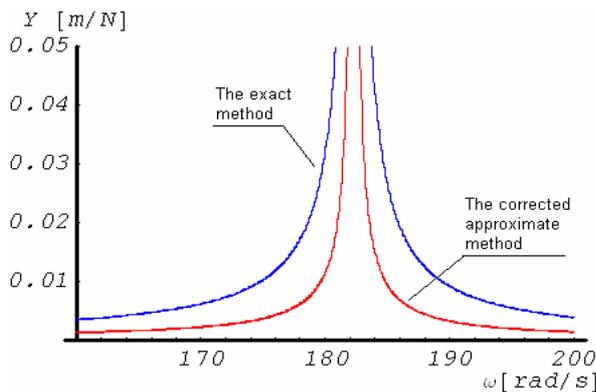


Fig. 9. The dynamic flexibility of the mechanical system, for $n=1$

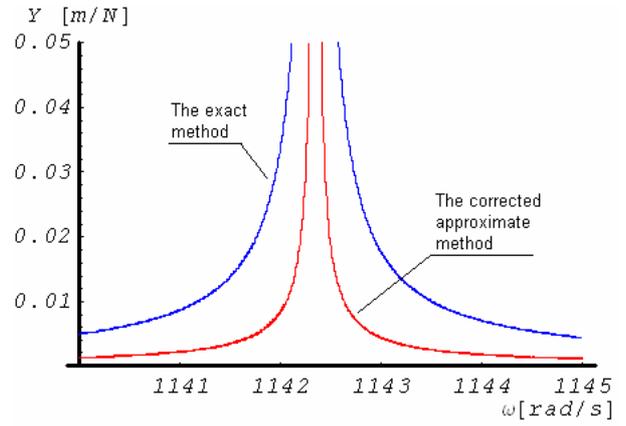


Fig. 10. The dynamic flexibility of the mechanical system, for $n=2$

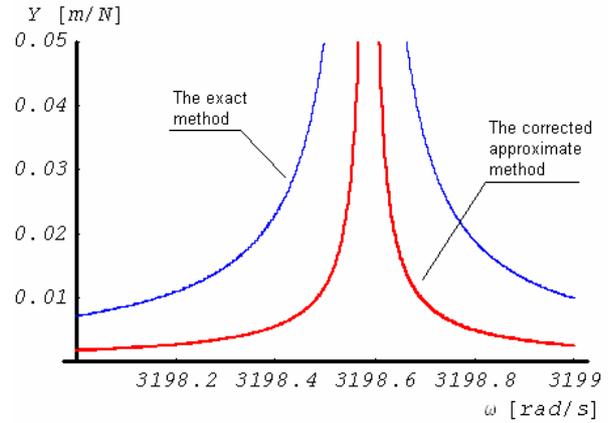


Fig. 11. The dynamic flexibility of the mechanical system, for $n=3$

6. THE MECHATRONIC SYSTEM

The corrected approximate Galerkin's method was used to assign the dynamic flexibility of the mechatronic system described by the discrete – continuous mathematical model. Obtained results, for the first three modes are presented in Fig. 12.

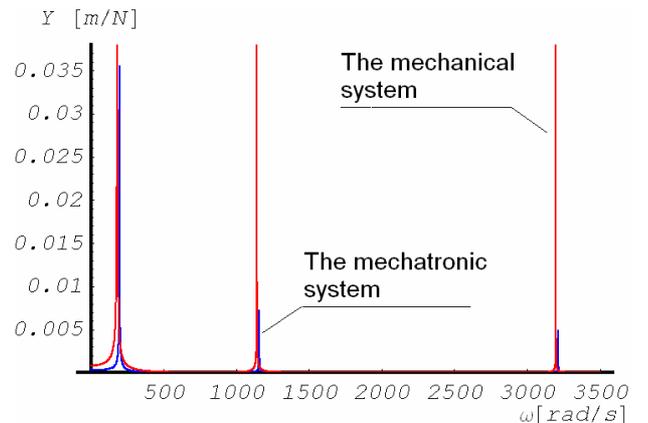


Fig. 12. The dynamic flexibility of the mechatronic system

In figures 13 to 15 obtained results can be presented more accurately. The beam with piezoelectric transducer (mechatronic system) is stiffer than the beam without it. What is more, implementation of the piezoelectric transducer with the external shunting

circuit results not only in limited values of the dynamic flexibility for all excitation frequencies, but also in shifting the resonance zone. Shift of the resonance zone depends on the system's geometrical and material parameters. For example if the glue layer becomes thinner and the value of the connection layer's modulus of elasticity in shear is increasing then the dynamic flexibility for all excitation frequencies have lower value and shifting of the resonance zone is increasing (see Fig 16 and 17). Influence of the piezoelectric constant and permittivity of the transducer material are shown in figures 18 and 19. Influence of the resistance of the external electric circuit is shown in figure 20.

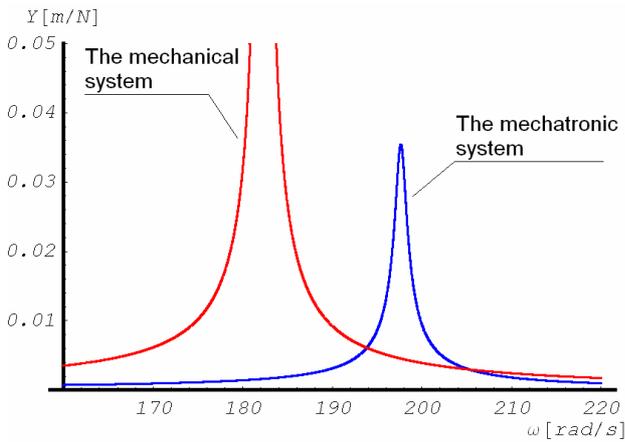


Fig. 13. The dynamic flexibility, for $n=1$

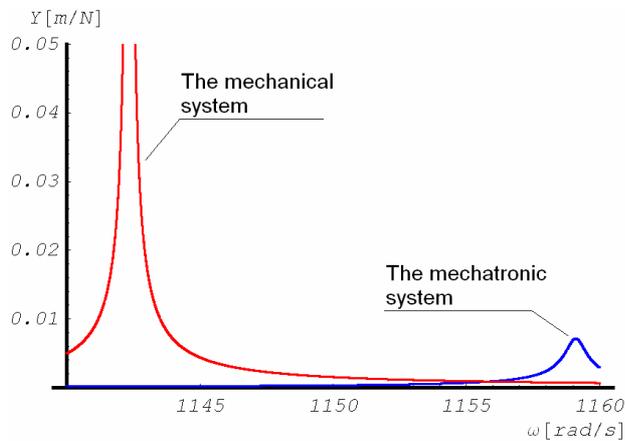


Fig. 14. The dynamic flexibility, for $n=2$

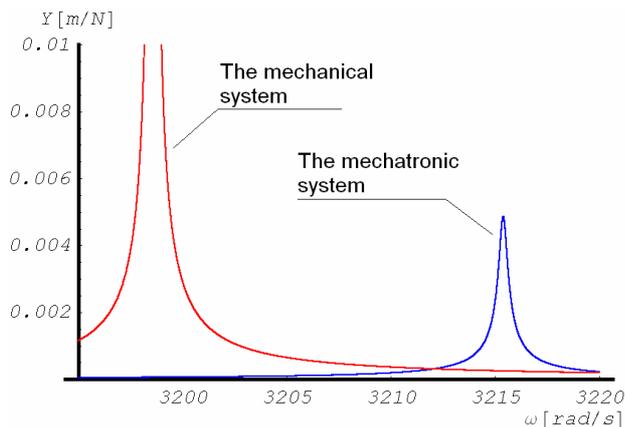


Fig. 15. The dynamic flexibility, for $n=3$

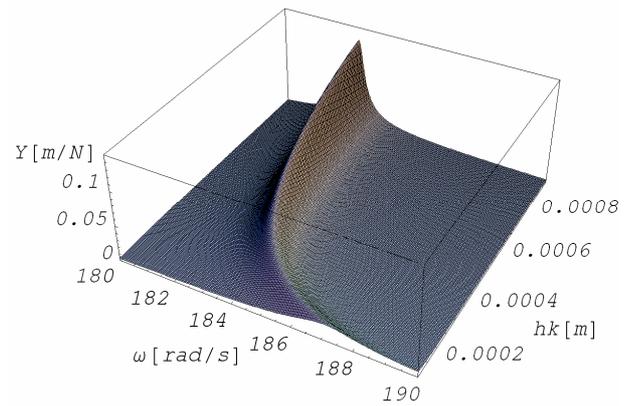


Fig. 16. Influence of the glue layer's thickness, for $n=1$

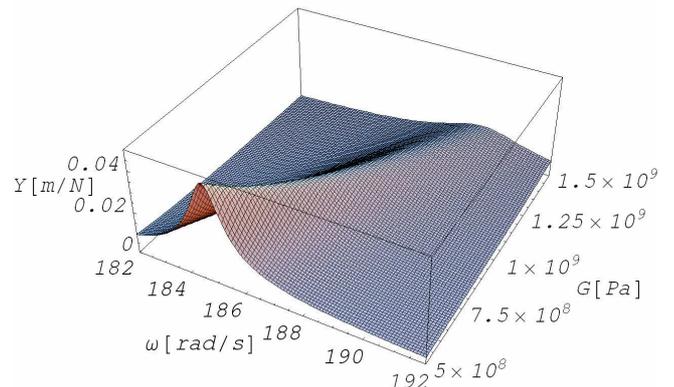


Fig. 17. Influence of the glue layer's modulus of elasticity in shear, for $n=1$

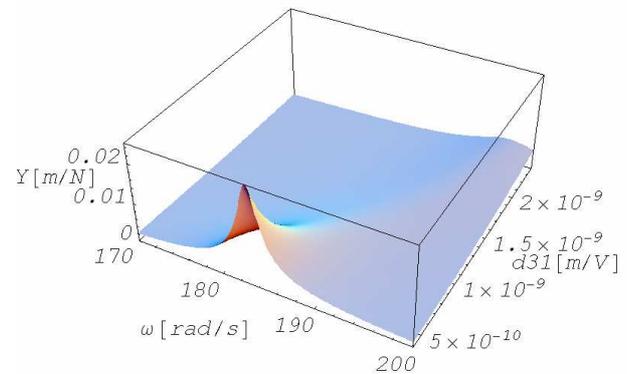


Fig. 18. Influence of the piezoelectric constant d_{31} of the transducer material, for $n=1$

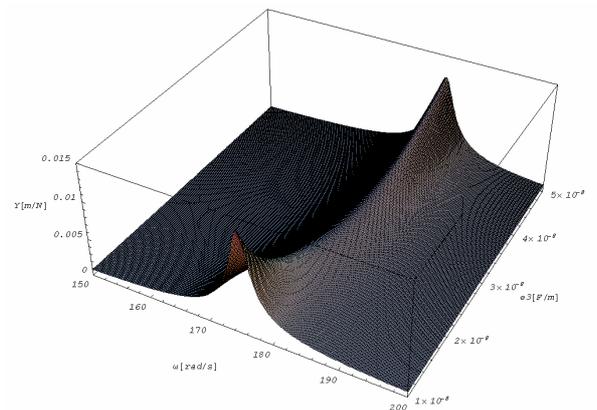


Fig. 19. Influence of the permittivity ϵ_{33} of the transducer material, for $n=1$

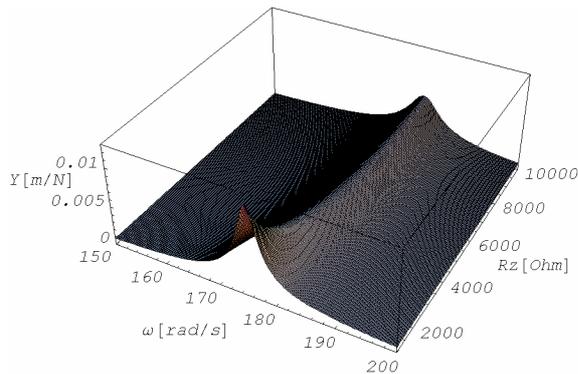


Fig. 20. Influence of the resistance of the external electric circuit, for $n=1$

7. CONCLUSIONS

Only very precise mathematical model of the mechatronic system enables the engineer to design a system with the required dynamic characteristic. Very precise method of the system's analysis is very important, too. It is impossible to use exact method relate to the mechatronic system's analysis, this is why the first step of the authors' work was to analyse the mechanical system, using the exact and approximate method to verify obtained results and decided if it can be used in such kind of analysis.

The approximate Galerkin's method with the correction coefficients introduced for the first three modes can be used to analyse the considered mechatronic system. Obtained results can be accepted and treat as very precise. The examination that was executed proved that the corrected approximate Galerkin's has very high accuracy and can be used in the further authors' work.

In previous authors' works it was proved, that the efficiency of damping of mechanical vibration depends on the beam, piezoelectric transducer and glue's material and geometrical parameters. Investigation with was made proved that there are optimum values of the external electric circuit's resistance and capacitance causes that the considered mechatronic system's dynamic flexibility has the lowest value.

8. ACKNOWLEDGEMENTS

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