

## CONTROL OF MECHANICAL SYSTEM DISPLACEMENT

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**Abstract:** The motion of pieces system takes place when a device is moving in a launcher or on a trajectory. General theorems of dynamics lead to the differential system of equations. The paper presents some aspects regarding the need of using some control systems to restrict the movement of an embarked mechanical system. In order to describe different response of the systems the authors have made several simulations both for the case with or without filter for an automatic system. Also, the paper tries to identify the response of the system for the case of an open loop and a close loop too.

**Key-Words:** mechanical embarked systems, filters, dynamics, open loop, close loop.

### 1. INTRODUCTION

Embarked pieces of rockets board apparatus are in motion during the systems displacement (Bălășoiu, 2001; Moraru, 2002).

The functions of these systems are put in connection with the reference time for a precise time computing or for other works.

Moreover, the functions are considered as safe properties for other systems when analyzing the motion stages. The characterization of the motion of any embarked pieces is a hard problem in the the study of the rocket motion.

Many safety-embarked pieces have a special form configuration and as a system, they have to respect a significant number of conditions, because the forces applied on are very small and to obtain good results, these pieces must be very sensitive (Bucur, 2004).

Yu et al. (2007) simulated the launch process of a ship-borne multiple launch rocket system; they used the exterior ballistic theory to characterize the rocket projectile flying.

Guilherme and Filho (2004) studied an active precession control system aiming to stop the precession motion of a sounding rocket, using an adequate strategy of displacement of the sensor and the actuator axes. In this paper, we shall treat the motion of embarked systems on rockets board, applying only the basic law of dynamics. The items

will be completed by graphical representations of motions and explanations.

### 2. THEORETICAL ASPECTS AND CASE STUDIES

Embarked systems shall be treated as rigid body objects, in accordance with the classic laws of kinematics and dynamics (Voinea et al., 1975, Cristescu, 1979). Mechanical systems can be used with control of motion, on the base of measuring embarked systems motion. In this way, the problem refers to ensuring displacement of mechanical pieces as embarked on high speed objects.

The most important fact in leading the piece motion is to have a feedback schema as shown in figure 1.

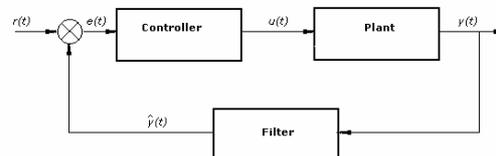


Fig. 1. Feedback schema

The action of the filter is described by a trajectory as presented in figure 2. We observe the line of true value, and also the response of filter tendencies to appropriates to the real value. The filter response is symbolized by  $\hat{y}(t)$  as predicted value, and the difference as error  $e(t)$  attached to the controller. Controller commands with  $u(t)$  signalize the plant to go to the real value of controlled parameter  $r(t)$ . The response of the plant is the signal  $y(t)$ . The signal  $y(t)$  is measured and enters the filter.

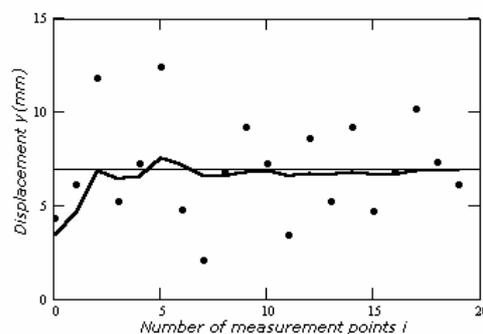


Fig.2. Displacement diagram

When the filter control system is not used, the feedback schema becomes that presented in figure 3.

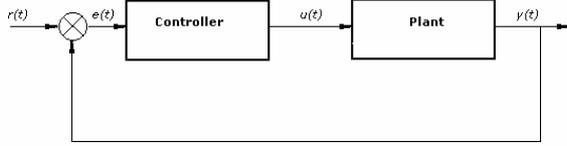


Fig. 3. Simplified feedback schema

The simplified feedback schema represents a classic control commander (Cristescu, 1979). In this case, we can write the differential equation of second order:

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = F(t), \quad (1)$$

where  $y(t)$  is the response of system and the force  $F(t)$  is the system input. To find the displacement relation, we must know the ratio of input and output:

$$\frac{y(t)}{F(t)} = \frac{1}{mD^2 + cD + k} \quad (2)$$

The derivative operator was symbolized by  $D$ . This leads to an expression similar to the Laplace transformation of second order:

$$ms^2Y(s) + csY(s) + kY(s) = F(s) \quad (3)$$

$$\frac{Y(s)}{F(s)} = \frac{1}{ms^2 + cs + k} \quad (4)$$

The diagram of the open loop response is shown in figure 4. The following parameters could be observed: *fast rise time*, when trajectory has about  $\pm 5\%$  deviation of output; *minimum overshoot* and *no steady-state error*.

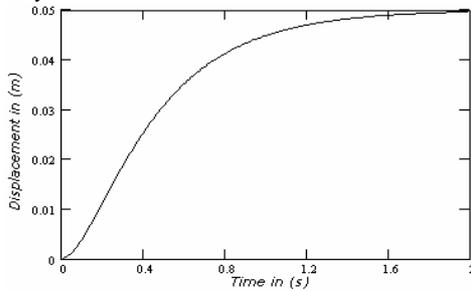


Fig. 4. Diagram of open loop response

For the first parameter (fig. 5), we can use a classic controller, similar to the home water regulator. The differences between the diagrams presented in figures 2 and 5 are due to the use a KF filter.

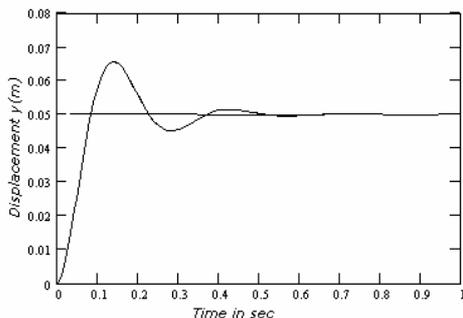


Fig. 5. Displacement diagram

Other type of diagram could be seen in figure 6. In this case, the system behavior is described by a derivative system with short *rise time*. The rise time is named time to obtain  $y(t)$  references, as shown in figures 1 and 3, where we defined the input  $r(t)$  as reference of the system with a tolerance of 5%.

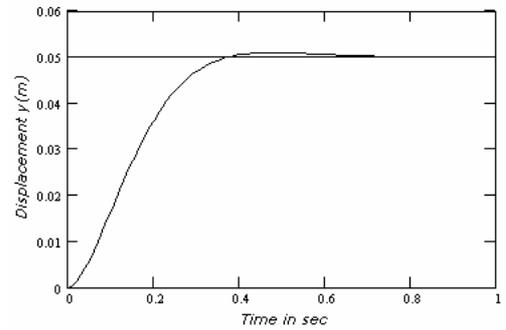


Fig. 6. Displacement diagram

The reference size input is  $r(t)=0,05$  and the controller gets up to normal size in 0,6 s, that is rise time much better than in the open loop case (fig. 4). The controller is defined as a proportional derivative, symbolized by PD. The filter touches  $r(t)$  with a great speed, as shown in figure 7.

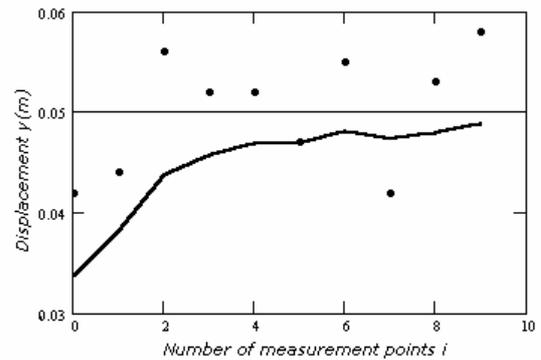


Fig. 7. Displacement diagram

In many cases of embarked systems, we need to combine mechanical with electric and electronic elements, for a safe motion of the systems. The mechanical pieces move in different conditions, specific to the high speed objects on which they are embarked (Bălăsoiu, 2001). The best controller is the PID (proportional integral derivative), which has a short rise time and no steady-error. Figure 8 presents a curve corresponding to the PID.

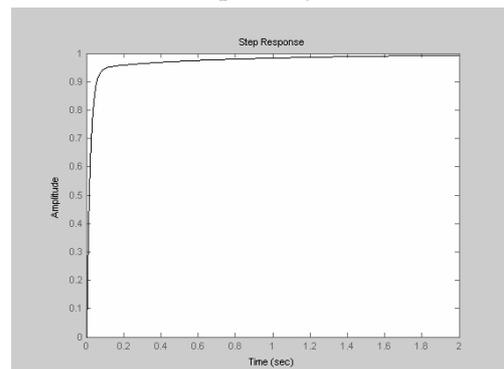


Fig. 8. PID curve

The advantages of this controller are the following: minimum rise time, absent overshoots, no steady error (this means a minimum difference between output and input). In figure 9, this error could be observed.

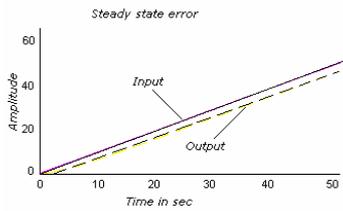


Fig. 9. Difference between output and input

The digital controllers based on AD/DA converters have a PID diagram as shown in figure 10.

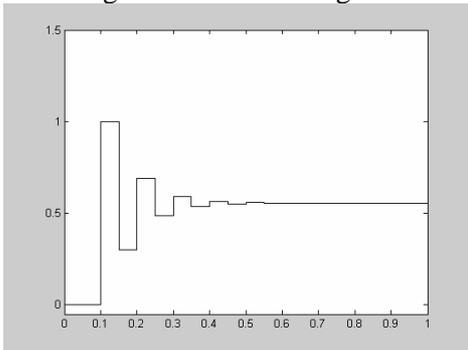


Fig. 10. PID diagram

In many cases, the embarked mechanical systems are characterized by second order differential equations. Thus, such a system with a mass body in cross displacement after  $Oy$  axis, with an elastic force  $F(t)$  as input, is presented in figure 11 (Moraru, 2002).

Mass motion is damped by the friction force  $2c\dot{y}$  and we can write the differential equation:

$$m\ddot{y} + 2c\dot{y} + ky = F(t) \quad (5)$$

$$t = t_0 \quad y(t_0) = 0 \quad \dot{y}(t_0) = 0$$

If we use the Laplace transformation, we will obtain:

$$ms^2Y(s) + 2csY(s) + kY(s) = F(s) \quad (6)$$

The ratio of output by input will be:

$$\frac{Y(s)}{F(s)} = \frac{1}{ms^2 + 2cs + k} \quad (7)$$

This is the famous relation determined many years ago. If we do not apply the Laplace transformation, we can use the derivative operator  $D$ , replacing the variable  $s$ .

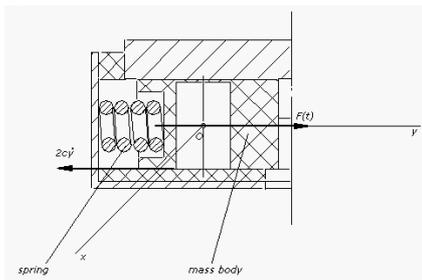


Fig. 11. Example of embarked mechanical system

We will try to analyze the case when  $m = 1 \text{ kg}$ ;  $F = 1N$ ;  $c = 0,1N \frac{s}{m}$ ;  $k = 0,1 \frac{N}{m}$ .

Using the equation (7) and the initial data, the formula becomes:

$$\frac{Y(s)}{F(s)} = \frac{1}{s^2 + 0,1s + 0,1} \quad (8)$$

The motion of body is illustrated in figure 4, but it is possible to obtain the wave form as proportional displacement, as result of different actions on the body. This influence is specific to the motion of high speed objects.

**Case 1 - open loop response.** In figure 12 we can see the curve corresponding to open loop response.

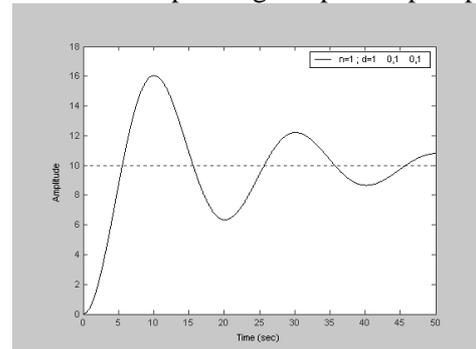


Fig. 12. Curve of open loop response

If we keep in mind the relevant notions like fast rise time and settling time, then 100 seconds represents a long time for using a system without control. The system is stable after hundreds seconds; therefore, we need to use a P controller with changes of coefficients in the transfer function. In this case, the transfer function is the following:

$$\frac{Y(s)}{F(s)} = \frac{1}{s^2 + 0.1s + 0.1} \quad (9)$$

The requirements to design a control system with PID controller are: a) rise time of about 0.05 seconds; b) settling time of 0.1 seconds; c) non overshoot; d) no steady error.

**Case 2 - closed loop representation.** We consider the implementation of a PID controller as shown in figure 13.

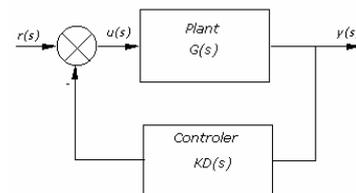


Fig. 13. Feedback schema

The curve corresponding to this controller is presented in figure 14. The transfer function can be written as:

$$\frac{Y(s)}{F(s)} = \frac{K_D s + K_P}{s^2 + (0.1 + K_D)s + (0.1 + K_P)} \quad (10)$$

The graphical representation shows a PD diagram with improved characteristics. After zooming, the diagram has the aspect presented in figure 15.

For settling time and overshoot, we will use a PID controller. In this case, the transfer function is:

$$\frac{Y(s)}{F(s)} = \frac{K_D s^2 + K_P s + K_I}{s^3 + (0.1 + K_D)s^2 + (0.1 + K_P)s + K_I} \quad (11)$$

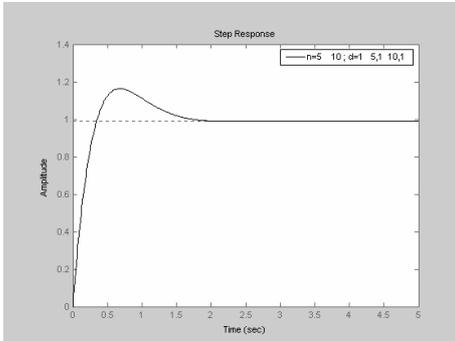


Fig. 14. Amplitude vs. time

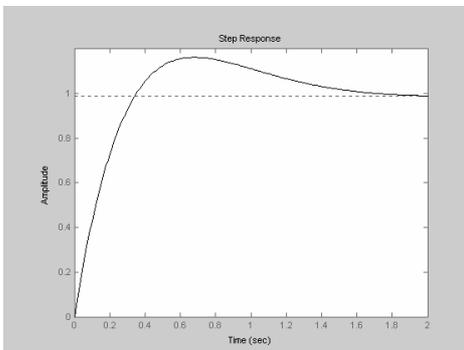


Fig. 15. Amplitude vs. time

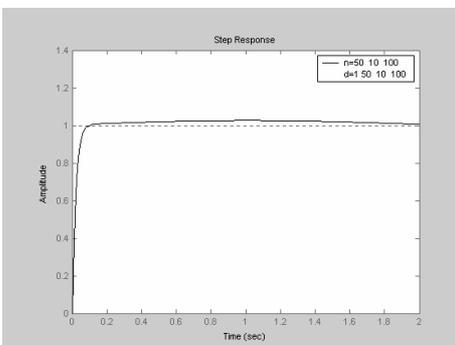


Fig. 16. Amplitude vs. time

If  $K_P = 10$ ,  $K_D = 50$ ,  $K_I = 100$ , we obtain the best controller. In zoom representation (fig. 17), we see better the effect of PID controller.

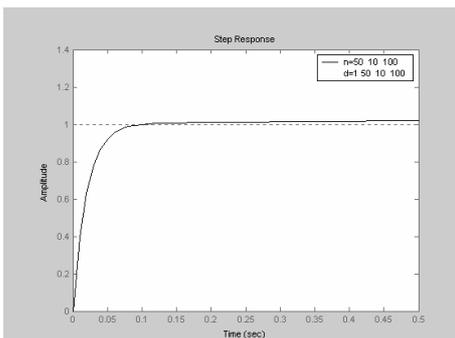


Fig. 17. Amplitude vs. time

The stability system is given in accordance with the Bode diagram. In figure 18, a plotted Bode diagram is presented.

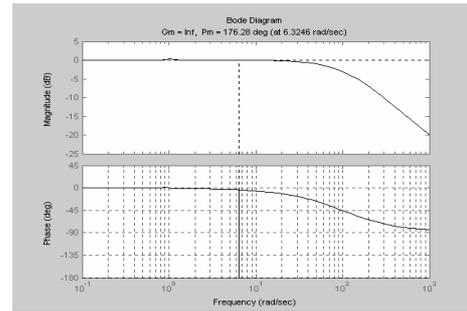


Fig. 18. Bode diagram

### 3. CONCLUSIONS

The present paperwork frames the theoretical works that study the motion of mechanical embarked systems.

Analysis of the response of an automatic system that use a PD or PID controller shows a number of differences in terms of time stabilization at the true value and in terms of time rise response.

Given the requirements of military systems for mechanical embarked systems a numerical simulation of the response of a specific fuze subsystem showed net benefits of implementing some type of PID controllers.

### 4. REFERENCES

1. Bălășoiu S. (2001) *Teoria sistemelor mecanice imbarcate*, Military Academy Publishing House, pp. 23-27, Bucharest, Romania
2. Bucur P., Postolea, D., Enache, C. (2004) *Mechanics. Embarked systems*, Pro Transilvania Publishing House, pp. 32-39, Bucharest, Romania
3. Cristescu R. (1979) *Functional Analyses*, Didactical and Pedagogical Publishing House, pp. 45-47, Bucharest, Romania
4. Guilherme, M.S., Filho, de W.C.L. (2004). *Active precession control system of a sounding rocket with control torque in only one axis*. In Advances in space dynamics 4: celestial mechanics and astronautics, Editor: H. K. Kuga, Instituto Nacional de Pesquisas Espaciais – INPE, São José dos Campos, SP, Brazil, pp. 161-169, ISBN 85-17-00012-9
5. Moraru F. (2002). *Numerical analysis charters. Applications for ballistics and flight dynamics*, Military Technical Academy, Bucharest, Romania
6. Voinea, R., Voiculescu, D., Ceausu V. (1975), *Mechanics*, Didactical and Pedagogical Publishing House, pp. 34-39, Bucharest, Romania
7. Yu, C., Ma, D., Zhang, X. (2007) *Launch process simulation of a ship-borne multiple launch rocket system*. World Journal of Modelling and Simulation, Vol. I, No. 1, pp. 58-65, ISSN 1 746-7233.

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