THE STATIONARY INCIDENT MOTION MODELING AROUND THE CIRCLE CALCULUS

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***Abstract*:** This article presents an approach of the aerodynamics study of the projectiles or missiles. It is part of a more extensive works on the influence of projectile shape on ballistic parameters.
The development of the phenomenon is helped by the development of the new CAD softwares. Using these softwares by specialists in the field, will improve the accuracy of the results obtained with less effort, especially in terms of resources.
Description phenomena rigorously using mathematical equations is the first step for use in determining the forces, the most powerful numerical method, the finite element method (FEM).
The paper proposes the transposition of the physical phenomenon into the mathematical equation in order to be easily simulated through these softwares.
Key words: flow regime, incident motion, subcritical speed

1. INTRODUCTION

The modern war theater requires a continuous development of weapons techniques in order to increase accuracy and efficiency. Worldwide major arms manufacturers use the most advanced manufacturing and testing technologies, results from the researches of scale. Regarding the missiles and reaction projectiles it seeks to improve their aerodynamic behavior in order to increase operating distance and improve accuracy. Software simulation and modeling are often used, but for this, the phenomena must be described very well in mathematical equations.
The modern weapons technologies become possible like the most used projectiles or missiles to have the head of projectile, ogive, built like a revolution body with big linear extensions ($\lambda \geq 3...4$), like conic surfaces, or made of steel surfaces with curved generators that meet certain criteria imposed.
The problem debated in this study refers at the influence of this conic surface on the entire missile aerodynamics.
Because it seeks to study the influence that one can have a removable conical surface, allowing a circle with three degrees of freedom (one translation and two rotations in relation with the Oxyz system bound with the carrier body), the basic pattern in the following will refer only to axially symmetric thin bodies with conical surface.
To study the subsonic motion without incidence in case of axially symmetrical conical bodies may be used the hypothesis of small perturbations.

2. PROBLEM FORMULATION

The motion has an incidence different of zero in the plane Ox1y1, the value of this angle must be small enough and constant in time. These initial conditions are imposed as restrictions for the next calculations

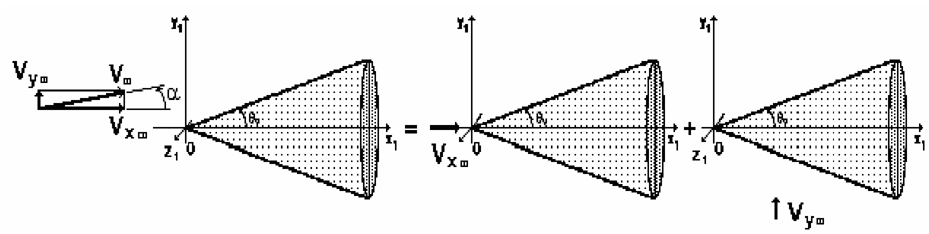

Fig. 1. Decomposition scheme for incident motion [2]

It can appreciate that the motion with incidende $\alpha \neq 0$, around a revolution body, if the incidence and the circle have radial symmetry, can be approximated like a motion without incidence and the motion of a transversal current with the same speed like the one of the initial current. All of these are illustrated in the figure 1.

The components of the speed into a cylindrical reference system are:

$$u' = V_{x_\infty} = V_\infty \cos\alpha ;$$
$$v'_r = V_\infty \sin\alpha \cos\omega ; \qquad (1)$$
$$v'_\omega = -V_\infty \sin\alpha \sin\omega .$$

There are illustrated in the figure 2.

The motion is well defined in a 3-D domain, so the flow can be determined completely.

The potential will be:

$$\Phi^* = x\,V_\infty \cos\alpha + \varphi + \Phi \qquad (2)$$

where φ is the potential due to the axial speed, $V_{x_\infty} = V_\infty \cos\alpha$, and Φ to the lateral speed, $V_{y_\infty} = V_\infty \sin\alpha$.

The potential from the relation (2) comply with the condition:

$$\left(1 - M_\infty^2\right)\frac{\partial^2\Phi^*}{\partial x^2} + \frac{\partial^2\Phi^*}{\partial y^2} + \frac{1}{y^2}\frac{\partial^2\Phi^*}{\partial\omega^2} + \frac{1}{y}\frac{\partial\Phi^*}{\partial y} = 0 \qquad (3)$$

where ω represents the angle of the median plan with the vertical reference plan that cross the symmetry axis of the solid, [1].

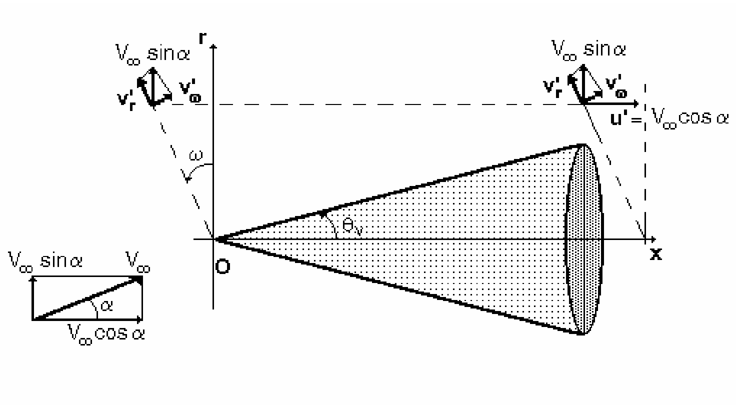

Fig. 2. Components of the speed into a cylindrical reference system [2]

3. PROBLEM SOLUTION

The study of the flow into a cross current has a lot of particularities regarding the dimensions of the solid and the variation of the cross section long of the *Ox* axis.

In this case the small perturbation method no longer provides satisfactory results.

The motion equations must be written in the complete form of them.

The equation of the pressure:

$$\frac{1}{2}V^2 + \frac{\gamma}{\gamma-1}\frac{p}{\rho} = \frac{\gamma}{\gamma-1}\frac{p_o}{\rho_o} = \frac{\gamma}{\gamma-1}RT_o = \text{const.} \qquad (4)$$

give the speed of sound:

$$a^2 = a_\infty^2 - \frac{\gamma-1}{2}\left(V^2 - V_\infty^2\right) \qquad (5)$$

where *V* is the speed in current point and V_∞ is the speed at the infinity.

Because we will study the side motion, we will retain the expression of potential from equation (2), so that the equation of motion will be:

$$\left[a_\infty^2 - \frac{\gamma-1}{2}\left(V^2 - V_\infty^2\right)\right]\Delta\Phi = \nabla\Phi \cdot \nabla\left(\frac{1}{2}V^2\right) \qquad (6)$$

We can get approximate solutions to the equation above, using either method of series development solution based on Mach number or iterations or numerical methods, seeking as the study of motion into a side stream to be modeled mathematically.

In order to study the lateral movement of the revolution body we can practice a selection, across section with a plane *P*, of an area of it, at a certain abscissa, *x*

The outline of the section in the plane *P* will be a circle of radius *r*, around which we can study the movement in polar coordinates, as can be seen in figure 3:

The potential motion around the circle in polar coordinates, is given by these equations:

$$\left[1 - \frac{\gamma-1}{2}M_\infty^2\left(\frac{V^2}{V_\infty^2} - 1\right)\right]\Delta\Phi =$$

$$= \frac{1}{2}\frac{M_\infty^2}{V_\infty^2}\left(\frac{\partial\Phi}{\partial r}\frac{\partial V^2}{\partial r} + \frac{1}{r^2}\frac{\partial\Phi}{\partial\omega}\frac{\partial V^2}{\partial\omega}\right) \qquad (7)$$

and

$$V^2 = \left(\frac{\partial\Phi}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\Phi}{\partial\omega}\right)^2 \qquad (8)$$

$$\Delta = \frac{\partial}{\partial r^2} + \frac{1}{r^2}\frac{\partial}{\partial\omega^2} + \frac{1}{r}\frac{\partial}{\partial r} \qquad (9)$$

where it was used only the potential function Φ, which define the lateral motion around the circular section of the solid (see the 2nd equation).

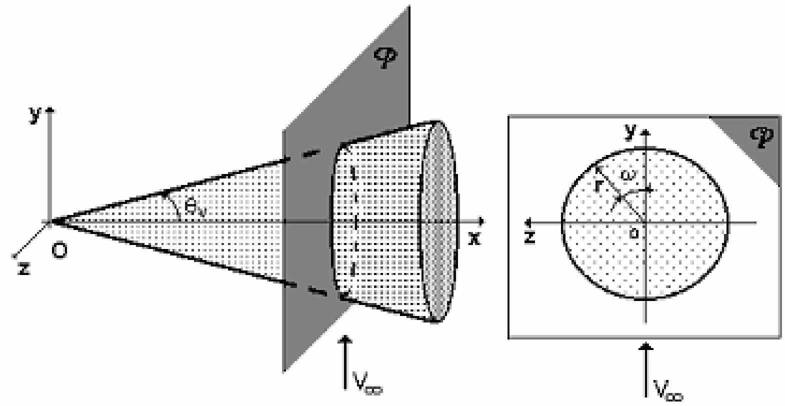

Fig. 3. description of motion in polar coordinates [2]

In order to obtain the circle motion, there are some restrictions that must be respected:

$$\frac{\partial\Phi}{\partial r} = 0 \text{, for } r=R;$$
$$\left(\frac{\partial\Phi}{\partial r}\right)_\infty = V_\infty \cos\omega;$$
$$\left(\frac{1}{r}\frac{\partial\Phi}{\partial\omega}\right)_\infty = -V_\infty \sin\omega. \quad (10)$$

The velocity V will be:

$$V^2 = q_0^2 + M_\infty^2 q_1^2 + M_\infty^4 q_2^2 + M_\infty^6 q_3^2 + \ldots, \quad (11)$$

using the following notations:

$$q_0^2 = \left(\frac{\partial\Phi_0}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\Phi_0}{\partial\omega}\right)^2; \quad (12)$$

$$q_1^2 = 2\left(\frac{\partial\Phi_0}{\partial r}\frac{\partial\Phi_1}{\partial r} + \frac{1}{r^2}\frac{\partial\Phi_0}{\partial\omega}\frac{\partial\Phi_1}{\partial\omega}\right); \quad (13)$$

$$q_2^2 = \left(\frac{\partial\Phi_1}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial\Phi_1}{\partial\omega}\right)^2 + 2\left(\frac{\partial\Phi_0}{\partial r}\frac{\partial\Phi_2}{\partial r} + \frac{1}{r^2}\frac{\partial\Phi_0}{\partial\omega}\frac{\partial\Phi_2}{\partial\omega}\right) \quad (14)$$

$$q_3^2 = 2\left[\frac{\partial\Phi_0}{\partial r}\frac{\partial\Phi_3}{\partial r} + \frac{\partial\Phi_1}{\partial r}\frac{\partial\Phi_2}{\partial r} + \ldots\right] 2\left[\ldots + \frac{1}{r^2}\left(\frac{\partial\Phi_0}{\partial\omega}\frac{\partial\Phi_3}{\partial\omega} + \frac{\partial\Phi_1}{\partial\omega}\frac{\partial\Phi_2}{\partial\omega}\right)\right] \quad (15)$$

it will obtain the following equation,

$$\left[1 - \frac{\gamma-1}{2}\frac{M_\infty^2}{V_\infty^2}\left(q_0^2 + M_\infty^2 q_1^2 + M_\infty^4 q_2^2 + \ldots - V_\infty^2\right)\right]\cdot$$
$$\left(\Delta\Phi_0 + M_\infty^2\Delta\Phi_1 + M_\infty^4\Delta\Phi_2 + \cdots\right) =$$
$$= \frac{M_\infty^2}{2V_\infty^2}\left[\left(\frac{\partial\Phi_0}{\partial r} + M_\infty^2\frac{\partial\Phi_1}{\partial r} + M_\infty^4\frac{\partial\Phi_2}{\partial r} + \ldots\right)\cdot\right.$$
$$\left(\frac{\partial q_0^2}{\partial r} + M_\infty^2\frac{\partial q_1^2}{\partial r} + M_\infty^4\frac{\partial q_2^2}{\partial r} + M_\infty^6\frac{\partial q_3^2}{\partial r} + \ldots\right) +$$
$$+ \frac{1}{r^2}\left(\frac{\partial\Phi_0}{\partial\omega} + M_\infty^2\frac{\partial\Phi_1}{\partial\omega} + \ldots\right) \ldots\ldots\ldots$$
$$\left.\ldots\ldots\ldots\left(\frac{\partial q_0^2}{\partial\omega} + M_\infty^2\frac{\partial q_1^2}{\partial\omega} + M_\infty^4\frac{\partial q_2^2}{\partial\omega} + M_\infty^6\frac{\partial q_3^2}{\partial\omega} + \ldots\right)\right] \quad (16)$$

that will lead on the identification of the specific terms:

$$\Delta\Phi_0 = 0;$$
$$\Delta\Phi_1 = \frac{1}{2V_\infty^2}\left(\frac{\partial\Phi_0}{\partial r}\frac{\partial q_0^2}{\partial r} + \frac{1}{r^2}\frac{\partial\Phi_0}{\partial\omega}\frac{\partial q_0^2}{\partial\omega}\right);$$
$$\Delta\Phi_2 = \frac{1}{2V_\infty^2}\left[\left(\frac{\partial\Phi_0}{\partial r}\frac{\partial q_1^2}{\partial r} + \frac{1}{r^2}\frac{\partial\Phi_0}{\partial\omega}\frac{\partial q_1^2}{\partial\omega}\right) + \ldots\ldots\right.$$
$$\left[\ldots\ldots + \left(\frac{\partial\Phi_1}{\partial r}\frac{\partial q_0^2}{\partial r} + \frac{1}{r^2}\frac{\partial\Phi_1}{\partial\omega}\frac{\partial q_0^2}{\partial\omega}\right) + \ldots\ldots\right.$$
$$\left. + \frac{\gamma-1}{V_\infty^2}\left(\frac{q_0^2}{V_\infty^2} - 1\right)\Delta\Phi_1\right];$$
$$\Delta\Phi_3 = \frac{1}{2V_\infty^2}\left[\left(\frac{\partial\Phi_0}{\partial r}\frac{\partial q_2^2}{\partial r} + \frac{1}{r^2}\frac{\partial\Phi_0}{\partial\omega}\frac{\partial q_2^2}{\partial\omega}\right) + \right.$$
$$+ \left(\frac{\partial\Phi_1}{\partial r}\frac{\partial q_1^2}{\partial r} + \frac{1}{r^2}\frac{\partial\Phi_1}{\partial\omega}\frac{\partial q_1^2}{\partial\omega}\right) +$$
$$+ \left(\frac{\partial\Phi_2}{\partial r}\frac{\partial q_0^2}{\partial r} + \frac{1}{r^2}\frac{\partial\Phi_2}{\partial\omega}\frac{\partial q_0^2}{\partial\omega}\right) +$$
$$\left. + (\gamma-1)\left(q_0^2 - V_\infty^2\right)\Delta\Phi_2 + (\gamma-1)q_1^2\Delta\Phi_1\right]. \quad (17)$$

knowing that,

$$\frac{\partial\Phi_0}{\partial r}=0\text{, pentru } r=R;\qquad \frac{\partial\Phi_1}{\partial r}=0\text{, pentru } r=R;\cdots$$

$$\left(\frac{\partial\Phi_0}{\partial r}\right)_\infty=V_\infty\cos\omega;\qquad \left(\frac{\partial\Phi_1}{\partial r}\right)_\infty=0;\cdots$$

$$\left(\frac{1}{r}\frac{\partial\Phi_0}{\partial\omega}\right)_\infty=-V_\infty\sin\omega;\qquad \left(\frac{1}{r}\frac{\partial\Phi_1}{\partial\omega}\right)_\infty=0;\cdots \tag{18}$$

and using the founded solution,

$$\Phi_0=V_\infty\left(r+\frac{R^2}{r}\right)\cos\omega \tag{19}$$

will obtain

$$q_0^2=V_\infty^2\left(1+\frac{R^4}{r^4}-2\frac{R^2}{r^2}\cos 2\omega\right) \tag{20}$$

and

$$\frac{\Delta\Phi_1}{V_\infty}=\left(-\frac{4R^4}{r^5}+\frac{2R^6}{r^7}\right)\cos\omega+\frac{2R^2}{r^3}\cos 3\omega \tag{21}$$

It consider the function f,

$$f=r^p\cos(m\omega) \tag{22}$$

Which satisfy the Poisson equation,

$$\Delta f=\left(p^2-m^2\right)r^{p-2}\cos(m\omega)\text{,} \tag{23}$$

and f is an harmonic if $p=\pm m$, then the solution of the equation (21) will be:

$$\frac{\Phi_1}{V_\infty}=\left(A\frac{R^2}{r}-\frac{R^4}{2r^3}+\frac{1}{12}\frac{R^6}{r^5}\right)\cos\omega+ +\left(-\frac{R^2}{4r}+B\frac{R^4}{r^3}\right)\cos 3\omega \tag{24}$$

Where it has been used the following harmonic solution $\frac{\cos\omega}{r},\frac{\cos 3\omega}{r^3}$

Using the results

$$A=\frac{13}{12},\ \mathrm{B}=\frac{1}{12}\text{ ,}$$

the solution Φ_1 will be like:

$$\frac{\Phi_1}{V_\infty}=\left(\frac{13}{12}\frac{R^2}{r}-\frac{1}{2}\frac{R^4}{r^3}+\frac{1}{12}\frac{R^6}{r^5}\right)\cos\omega+ +\left(-\frac{1}{4}\frac{R^2}{r}+\frac{1}{12}\frac{R^4}{r^3}\right)\cos 3\omega \tag{25}$$

In the same way it will obtain the solution Φ_2 and Φ_3, the last one will show the influence of the high rank terms on the air compressibility.

For Φ_2

$$\frac{\Phi_2}{V_\infty}=\Phi_{21}+(\gamma-1)\Phi_{22} \tag{26}$$

where it has been used the notations [5]:

$$\Phi_{21}=\left(\frac{623}{320}\frac{R^2}{r}-\frac{87}{64}\frac{R^4}{r^3}+\frac{25}{32}\frac{R^6}{r^5}-\frac{5}{16}\frac{R^8}{r^7}+\frac{11}{240}\frac{R^{10}}{r^9}\right)\cos\omega+ +\left(-\frac{7}{48}\frac{R^2}{r}-\frac{83}{432}\frac{R^4}{r^3}+\frac{3}{16}\frac{R^6}{r^5}-\frac{1}{24}\frac{R^8}{r^7}+\frac{1}{144}\frac{R^{10}}{r^9}\right)\cos 3\omega+ +\left(\frac{1}{16}\frac{R^2}{r}+\frac{1}{16}\frac{R^4}{r^3}-\frac{1}{20}\frac{R^6}{r^5}\right)\cos 5\omega\text{ ,} \tag{26a}$$

$$\Phi_{22}=\left(\frac{17}{60}\frac{R^2}{r}-\frac{1}{8}\frac{R^4}{r^3}+\frac{1}{12}\frac{R^6}{r^5}-\frac{1}{16}\frac{R^8}{r^7}+\frac{1}{80}\frac{R^{10}}{r^9}\right)\cos\omega+ +\left(-\frac{61}{240}\frac{R^4}{r^3}+\frac{3}{16}\frac{R^6}{r^5}-\frac{1}{40}\frac{R^8}{r^7}\right)\cos 3\omega+ +\left(\frac{1}{16}\frac{R^4}{r^3}-\frac{3}{80}\frac{R^6}{r^5}\right)\cos 5\omega \tag{26b}$$

and for Φ_3

$$\frac{2}{V_\infty}\Phi_3=\Phi_{31}+(\gamma-1)\Phi_{32}+(\gamma-1)^2\Phi_{33} \tag{27}$$

where:

$$\Phi_{31}=\left(-\frac{95879}{100800}\frac{R^2}{r}-\frac{22597}{11520}\frac{R^4}{r^3}+\frac{205271}{34560}\frac{R^6}{r^5}- -\frac{385313}{69120}\frac{R^8}{r^7}+\frac{63913}{23040}\frac{R^{10}}{r^9}-\frac{1943}{2400}\frac{R^{12}}{r^{11}}+\frac{77}{10080}\frac{R^{14}}{r^{13}}\right)\cos\omega+ +\left(\frac{239}{1920}\frac{R^2}{r}-\frac{1517357}{806400}\frac{R^4}{r^3}+\frac{75983}{23040}\frac{R^6}{r^5}-\frac{1051}{450}\frac{R^8}{r^7}+ +\frac{25781}{34560}\frac{R^{10}}{r^9}-\frac{29}{224}\frac{R^{12}}{r^{11}}+\frac{1}{144}\frac{R^{14}}{r^{13}}\right)\cos 3\omega+ +\left(\frac{13}{192}\frac{R^2}{r}+\frac{1105}{1152}\frac{R^4}{r^3}-\frac{173767}{241920}\frac{R^6}{r^5}+\frac{725}{6912}\frac{R^8}{r^7}-\frac{1}{133}\frac{R^{10}}{r^9}- -\frac{1}{144}\frac{R^{12}}{r^{11}}+\frac{1}{864}\frac{R^{14}}{r^{13}}\right)\cos 5\omega+ +\left(-\frac{1}{32}\frac{R^2}{r}-\frac{3}{16}\frac{R^4}{r^3}+\frac{1}{12}\frac{R^6}{r^5}+\frac{17}{672}\frac{R^8}{r^7}\right)\cos 7\omega \tag{27a}$$

$$\Phi_{32}=\left(-\frac{7753}{4032}\frac{R^2}{r}-\frac{197}{320}\frac{R^4}{r^3}+\frac{9643}{5760}\frac{R^6}{r^5}-\frac{9043}{5760}\frac{R^8}{r^7}+ +\frac{343}{320}\frac{R^{10}}{r^9}-\frac{7}{20}\frac{R^{12}}{r^{11}}+\frac{153}{3360}\frac{R^{14}}{r^{13}}\right)\cos\omega+ +\left(-\frac{17}{120}\frac{R^2}{r}-\frac{36213}{89600}\frac{R^4}{r^3}+\frac{31}{12}\frac{R^6}{r^5}-\frac{16247}{9600}\frac{R^8}{r^7}+\frac{401}{720}\frac{R^{10}}{r^9}- -\frac{123}{1120}\frac{R^{12}}{r^{11}}+\frac{1}{120}\frac{R^{14}}{r^{13}}\right)\cos 3\omega+ +\left(\frac{41}{60}\frac{R^4}{r^3}-\frac{1037}{5600}\frac{R^6}{r^5}-\frac{5}{24}\frac{R^8}{r^7}+\frac{53}{1120}\frac{R^{10}}{r^9}-\frac{1}{120}\frac{R^{12}}{r^{11}}\right)\cdot \cos 5\omega+\left(-\frac{1}{8}\frac{R^4}{r^3}-\frac{1}{96}\frac{R^6}{r^5}+\frac{41}{672}\frac{R^8}{r^7}\right)\cos 7\omega \tag{27b}$$

$$\Phi_{33}=\left(-\frac{377}{840}\frac{R^2}{r}+\frac{1}{48}\frac{R^6}{r^5}-\frac{1}{16}\frac{R^8}{r^7}+\frac{3}{80}\frac{R^{10}}{r^9}-\right.$$
$$\left.-\frac{1}{60}\frac{R^{12}}{r^{11}}+\frac{1}{336}\frac{R^{14}}{r^{13}}\right)\cos\omega+$$
$$+\left(-\frac{29}{210}\frac{R^4}{r^3}+\frac{1}{16}\frac{R^6}{r^5}-\frac{1}{40}\frac{R^8}{r^7}+\frac{1}{24}\frac{R^{10}}{r^9}-\frac{1}{112}\frac{R^{12}}{r^{11}}\right)\cos 3\omega+$$
$$+\left(\frac{169}{1680}\frac{R^6}{r^5}-\frac{1}{12}\frac{R^8}{r^7}+\frac{1}{112}\frac{R^{10}}{r^9}\right)\cos 5\omega+$$
$$+\left(-\frac{1}{48}\frac{R^6}{r^5}+\frac{5}{168}\frac{R^8}{r^7}\right)\cos 7\omega \qquad (27c)$$

The relations above mentioned allow the calculus of the coefficients of the serial development of the velocity for the circular section:

$$\frac{1}{V_\infty^2}q_0^2=2-2\cos 2\omega \qquad (28)$$

$$\frac{1}{V_\infty^2}q_1^2=\frac{4}{3}-\frac{7}{3}\cos 2\omega+\cos 4\omega \qquad (29)$$

$$\frac{1}{V_\infty^2}q_2^2=-2{,}440278+1{,}627778\cos 2\omega+1{,}6875\cos 4\omega+$$
$$-0{,}875\cos 6\omega+(\gamma-1).$$
$$.(0{,}625-1{,}175\cos 2\omega+0{,}8\cos 4\omega-0{,}25\cos 6\omega) \qquad (30)$$

$$\frac{1}{V_\infty^2}q_3^2=-1{,}423928+1{,}425736\cos 2\omega+2{,}245919\cos 4\omega-$$
$$-3{,}20606\cos 6\omega+0{,}958333\cos 8\omega+$$
$$+(\gamma-1)(-1.401637+3{,}599163\cos 2\omega-$$
$$-0{,}395145\cos 4\omega-2{,}385714\cos 6\omega+0{,}583333\cos 8\omega)+$$
$$+(\gamma-1)^2(-0{,}466667+0{,}263096\cos 2\omega+0{,}334523\cos 4\omega-$$
$$-0{,}068452\cos 6\omega-0{,}0625\cos 8\omega) \qquad (31)$$

The velocity on the circle will be:

$$V^2=q_0^2+M_\infty^2q_1^2+M_\infty^4q_2^2+M_\infty^6q_3^2+\ldots \qquad (32)$$

or using the equation,

$$-\frac{V}{V_\infty}=-2\sin\omega+M_\infty^2\left(-\frac{2}{3}\sin\omega+\frac{1}{2}\sin 3\omega\right)+$$
$$+M_\infty^4\left[-\frac{1058}{960}\sin\omega+\frac{5}{9}\sin 3\omega-\frac{3}{8}\sin 5\omega+(\gamma-1)\left(-\frac{233}{240}\sin\omega+\right.\right.$$
$$\left.\left.+\frac{11}{40}\sin 3\omega-\frac{1}{8}\sin 5\omega\right)\right]+$$
$$+M_\infty^6[0{,}2878\sin\omega+0{,}2571\sin 3\omega-1{,}0005\sin 5\omega+0{,}3854\sin 7\omega+$$
$$+(\gamma-1)(0{,}8335\sin\omega-1{,}2008\sin 3\omega-0{,}822\sin 5\omega+0{,}26\sin 7\omega)+$$
$$+(\gamma-1)^2(0{,}233\sin\omega+0{,}102\sin 3\omega-0{,}066\sin 5\omega-0{,}031\sin 7\omega)] \qquad (33)$$

4. CONCLUSIONS

For the interval $0\le\omega\le\pi$, the maximum speed will be obtained for $\omega=\frac{\pi}{2}$. The variation of the function $\frac{V}{V_\infty}$ given by the relation (33) is presented in the figure 4.
The calculus has been made without the terms that include the physical aspect of the compressibility (1). In the other curves, the terms M_∞^2 (3), M_∞^4 (2) or M_∞^6 (4) are used in order to approximate the real physical aspect.

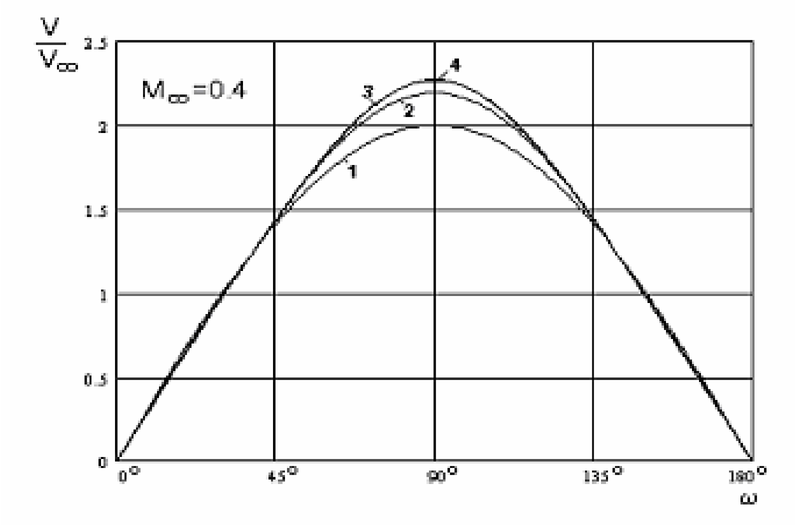

Fig. 4. Variation of the function $\frac{V}{V_\infty}$ [4]

In the figure 5 it can be seen that the term M_∞^6 makes that for the function $\frac{V}{V_\infty}$ the maximum value to be 2,2706, for: $M_\infty = 0{,}4081$, and $\gamma = 1{,}40\tilde{5}$.

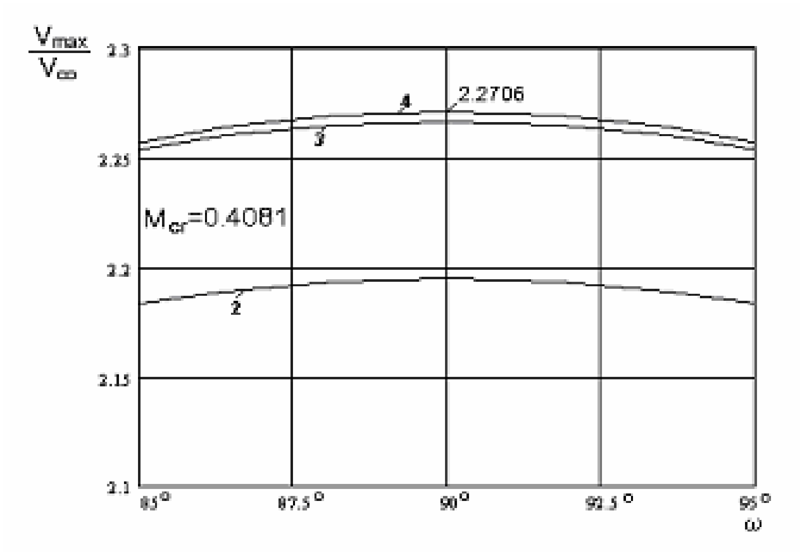

Fig. 5. Variation of the function $\frac{V}{V_\infty}$ [4]

All the terms of the equation has been taken into the calculus, no matter of their algebraic sign.
It can be seen that the upper degree solutions effect can be neglected only if M_∞ <<1.
The term M_∞^6 has a contribution of (4 ÷ 7) %, for subcritical speeds.
The flow remains subsonic if the number M_∞ is lower than the critical number Mach that can be determined from the equation [3]:

$$\frac{a^2}{V_\infty^2} = \frac{1}{M_\infty^2}\left[1 - \frac{\gamma - 1}{2} M_\infty^2 \left(\frac{V^2}{V_\infty^2} - 1\right)\right] \tag{34}$$

where $V_{max} = a = c$, when $M_\infty = M_{cr}$:

$$\frac{V_{\max}^2}{V_\infty^2} = \frac{1}{M_{cr}^2}\left[1 - \frac{\gamma - 1}{2} M_{cr}^2 \left(\frac{V_{\max}^2}{V_\infty^2} - 1\right)\right] \tag{35}$$

The value of the critical Mach number, M_{cr} has been determined by the successive approximations:

$$M_{cr} = 0{,}4081. \tag{36}$$

It can see that the value of the critical Mach number is relative small and it will influence the flow regime. To determine the maximum speed the equation will have the form:

$$\frac{V_{\max}}{V_\infty} = 2 + 1{,}1667\, M_\infty^2 + 2{,}5878\, M_\infty^4 + 0{,}9537\, M_\infty^6 + \cdots. \tag{37}$$

5. REFERENCES

1. Avram E., Nastasescu V., Malos G., (2005) *Metoda Elementelor finite în mecanica fluidelor*, Military Technical Academy Publishing House, Bucharest, Romania
2. Barbu C., (2005) *Aerodinamica Computațională*, Military Technical Academy Publishing House, Bucharest, Romania
3. Chereches T., Sava A.C., (2008). *Some aspects regarding the using of the dynamics arrays in the numerical simulation of the hydraulic and pneumatic devices*, Proceedings of the 6th IASME/WSEAS International Conferfence on Fluid Mechanics and Aerodynamics (FMA 08), pp. 264-268, ISSN 1790-5095, ISBN 978-960-6766-98-5, WSEAS Press, Rhodes, Greece.
4. Gavrilă J., (2005) *Mecanica şi aerodinamica aparatelor de zbor*, Military Technical Academy Publishing House, Bucharest, Romania
5. Sava A.C., Chereches T., (2008). *Modeling around the circle of the stationary incident motion*, Proceedings of the 6th IASME/WSEAS International Conferfence on Fluid Mechanics and Aerodynamics (FMA 08), pp. 259-263, ISSN 1790-5095, ISBN 978-960-6766-98-5, WSEAS Press, Rhodes, Greece.

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